DIRECTORATE OF DISTANCE EDUCATION

UNIVERSITY OF NORTH BENGAL

MASTER OF ARTS- PHILOSOPHY

SEMESTER –II

WESTERN LOGIC

CORE-201

BLOCK-1

UNIVERSITY OF NORTH BENGAL

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ENLIGHTENMENT TO PERFECTION

FOREWORD

The Self Learning Material (SLM) is written with the aim of providing simple and organized study content to all the learners. The SLMs are prepared on the framework of being mutually cohesive, internally consistent and structured as per the university's syllabi. It is a humble attempt to give glimpses of the various approaches and dimensions to the topic of study and to kindle the learner's interest to the subject

We have tried to put together information from various sources into this book that has been written in an engaging style with interesting and relevant examples. It introduces you to the insights of subject concepts and theories and presents them in a way that is easy to understand and comprehend.

We always believe in continuous improvement and would periodically update the content in the very interest of the learners. It may be added that despite enormous efforts and coordination, there is every possibility for some omission or inadequacy in few areas or topics, which would definitely be rectified in future.

We hope you enjoy learning from this book and the experience truly enrich your learning and help you to advance in your career and future endeavours.

WESTERN LOGIC

BLOCK-1

BLOCK-2

BLOCK 1 : WESTERN LOGIC

Introduction to the Block

Unit 1 deals with introducing and familiarizing the definition, nature and scope of the subject exposing the students to various definitions of logic and discussing the question whether it is an art or a science, a positive science or a normative science.

Unit 2 deals with the team Logic whch is said to be the study of argument as expressed in language. Language in general is highly ambiguous. In any language words are often used in various senses. For example, sometimes ‘thought’ and knowledge’ are used as synonymous terms.

Unit 3 deals with thought and thoughts are always expressed in language in which different words we use are expected to convey proper idea. If there are no fixed ideas, it would be difficult to understand what one means by a word.

Unit 4 how the truth-functional concepts of negation, conjunction, disjunction, material conditionality, and material biconditionality may be expressed in English as well as in symbols

Unit 5 deals with introduce you to the concept of equivalence through two means; truth table method and stroke and dagger function and contradiction through truth-table means.

Unit 6 deals with any compound proposition to determine its truth-value and symbolic representation of statements helps better understanding than verbal representation which is not only more complicated in structure but also ambiguous

A unit 7 deal with an attempt is made to present a history of symbolic logic and to notice that the moment you enter symbolic logic, you are confronted with mathematics as well.

UNIT 1: NATURE AND SCOPE OF LOGIC

STRUCTURE

1.0 OBJECTIVES

This unit titled Nature and Scope of Logic aims at:

- To introducing and familiarizing the definition, nature and scope of the subject exposing the students to various definitions of logic.
- To discussing the question whether it is an art or a science, a positive science or a normative science
- To discussing the extension and scope of logic

1.1 INTRODUCTION

"Reasons are the coin we pay for the belief we hold," so says Schipper in his monumental work on Model logic. But reasons given are not always good enough. With reasoning we produce arguments – some good, some bad – that often get converted in writing. Every argument confronted raises this question: Does the conclusion reached follow from the premises used or assumed? There are objective criteria with which that question can be answered, in the study of logic we seek to discover and apply those criteria. Usually logic is associated with Greek tradition and philosophy. Most of us think logic as a branch of knowledge originated in ancient Greece. But this is not true since as a matter of fact almost all great civilizations developed logic as an academic discipline. Ancient Indians, Arabs, and Chinese made significant contributions to the growth and development of logic. However, our study is restricted logic developed by Europeans over several centuries.

Logic as a Discipline

Nature and varieties of logic

It is relatively easy to discern some order in the above embarrassment of explanations. Some of the characterizations are in fact closely related to each other. When logic is said, for instance, to be the study of the laws of thought, these laws cannot be the empirical (or observable) regularities of actual human thinking as studied in psychology; they must be laws of correct reasoning, which are independent of the psychological idiosyncrasies of the thinker. Moreover, there is a parallelism between correct thinking and valid argumentation: valid argumentation may be thought of as an expression of correct thinking, and the latter as an internalization of the former. In the sense of this parallelism, laws of correct thought will match those of correct argumentation. The characteristic mark of the latter is, in turn, that they do not depend on any particular matters of fact. Whenever an argument that takes a reasoner from p to q is valid, it must hold independently of what he happens to know or believe about the subject matter of p and q. The only other source of the certainty of the connection between p and q, however, is presumably constituted by the meanings of the terms that the propositions p and q contain. These very same meanings will then also make the sentence “If p, then q” true irrespective of all contingent matters of fact. More generally, one can validly argue from p to q if and only if the implication “If p, then q” is logically true—i.e., true in virtue of the meanings of words occurring in p and q, independently of any matter of fact.

The following proposition (from Aristotle), for instance, is a simple truth of logic: "If sight is perception, the objects of sight are objects of perception." Its truth can be grasped without holding any opinions as to what, in fact, the relationship of sight to perception is. What is needed is merely an understanding of what is meant by such terms as "if–then," "is," and "are," and an understanding that "objects of" expresses some sort of relation.

The logical truth of Aristotle's sample proposition is reflected by the fact that "The objects of sight are objects of perception" can validly be inferred from "Sight is perception."

Many questions nevertheless remain unanswered by this characterization. The contrast between matters of fact and relations between meanings that was relied on in the characterization has been challenged, together with the very notion of meaning. Even if both are accepted, there remains a considerable tension between a wider and a narrower conception of logic. According to the wider interpretation, all truths depending only on meanings belong to logic. It is in this sense that the word logic is to be taken in such designations as "epistemic logic" (logic of knowledge), "doxastic logic" (logic of belief), "deontic logic" (logic of norms), "the logic of science," "inductive logic," and so on. According to the narrower conception, logical truths obtain (or hold) in virtue of certain specific terms, often called logical constants. Whether they can be given an intrinsic characterization or whether they can be specified only by enumeration is a moot point. It is generally agreed, however, that they include (1) such propositional connectives as "not," "and," "or," and "if–then" and (2) the so-called quantifiers "(∃x)" (which may be read: "For at least one individual, call it x, it is true that") and "(∀x)" ("For each individual, call it x, it is true that"). The dummy letter x is here called a bound (individual) variable. Its values are supposed to be members of some fixed class of entities, called individuals, a class that is variously known as the universe of discourse, the universe presupposed in an interpretation, or the domain of individuals. Its members are said to be quantified over in “(∃x)” or “(∀x).” Furthermore, (3) the concept of identity (expressed by =) and (4) some notion of predication (an individual’s having a property or a relation’s holding between several individuals) belong to logic. The forms that the study of these logical constants takes are described in greater detail in the article logic, in which the different kinds of logical notation are also explained. Here, only a delineation of the field of logic is given.

When the terms in (1) alone are studied, the field is called propositional logic. When (1), (2), and (4) are considered, the field is the central area of logic that is variously known as first-order logic, quantification theory, lower predicate calculus, lower functional calculus, or elementary logic. If the absence of (3) is stressed, the epithet “without identity” is added, in contrast to first-order logic with identity, in which (3) is also included.

Borderline cases between logical and nonlogical constants are the following (among others): (1) Higher order quantification, which means quantification not over the individuals belonging to a given universe of discourse, as in first-order logic, but also over sets of individuals and sets of n-tuples of individuals. (Alternatively, the properties and relations that specify these sets may be quantified over.) This gives rise to second-order logic. The process can be repeated. Quantification over sets of such sets (or of n-tuples of such sets or over properties and relations of such sets) as are considered in second-order logic gives rise to third-order logic; and all logics of finite order form together the (simple) theory of (finite) types. (2) The membership relation, expressed by ∊, can be grafted on to first-order logic; it gives rise to set theory. (3) The concepts of (logical) necessity and (logical) possibility can be added.

This narrower sense of logic is related to the influential idea of logical form. In any given sentence, all of the nonlogical terms may be replaced by variables of the appropriate type, keeping only the logical constants intact. The result is a formula exhibiting the logical form of the sentence.

If the formula results in a true sentence for any substitution of interpreted terms (of the appropriate logical type) for the variables, the formula and the sentence are said to be logically true (in the narrower sense of the expression).

1.2 VARIOUS DEFINITIONS OF LOGIC

The word 'logic' comes from the Greek word logos, literally meaning, word, thought, speech, reason, energy and fire. But in due course of time these literal meanings were given up to make way for more accurate meaning hinting at what we actually learn when we do logic. This is how it came to be understood as a discipline dealing with thought, reasoning and argument at different points of time. It is our experience that emotional appeal is sometimes effective. But it has no place in logic. Only appeal to reason pays effectively in the long run and which can be objectively verified and appraised. One needs to discern the criteria involved in rational method. The goal of the study of logic is to discover and make available those criteria that can be used to test the correctness of arguments. Against this background we shall evaluate various definitions of logic held at different times and their merits and demerits. One of the definitions of logic states that it is the study of reflective thinking. This particular definition was proposed by Susan Stebbing in her work 'A Modern Introduction to Logic'. She, surely, made progress over H.W.B. Joseph who regarded thought in its unqualified sense as the main theme of logic when he wrote 'Introduction to Logic'. However, the fact is that one has to concede in both the cases that the content of logic is essentially psychological and what is psychological is invariably subjective. This position is unacceptable to any student of logic. A clarification is needed on this issue. One of the important topics of logic is what is known as 'Laws of Thought.' There are three laws of thought, law of identity, law of excluded middle and law of of contradiction. On this ground, it is possible to conclude that at least indirectly logic deals with thought. However, this is a mistaken notion. Laws of thought, in reality, have nothing to do with thought.

They merely show or demonstrate the nature of statements. Therefore even in this sense thought cannot enter the domain of logic. Another discarded definition of logic states that it is the study of the methods or principles which we use to distinguish good (correct) reasoning from bad (incorrect) reasoning. As it has been claimed ‘All reasoning is thinking but all thinking is not reasoning’. There are many psychological processes that are different from reasoning, such as imagining, regretting, day dreaming and so on. There seems to be same laws governing all these activities, but they are not studied by logicians. Reasoning is a special kind of thinking in which problems are solved and conclusions are drawn from premises. The logician is primarily concerned with the correctness of the completed process of reasoning and only with this species of thinking. This definition does not imply that only a student of logic can reason well. Nor does it imply that a student of logic necessarily does it. Just as an athlete need not be aware of the complex processes going on inside his body while he performs the athletic fete, people need not be conscious of the complex logical processes involved in reasoning when they scrupulously perform the task of reasoning. However, a person, who has studied logic, is more likely (there is no rule that he should do) to reason correctly than one who has never thought about the principles involved in logical activity.

There are multiple reasons for it. To begin with, a student of logic will approach the discipline as an art as well as a science, and he or she will engage herself in doing exercises in all parts of the theory being learned. It is a continuous practice that will help the student fare better and make him perfect. Second, a significant part of the study of logic consists in the examination and analysis of fallacies, which may be viewed as quite natural mistakes in reasoning. Knowledge of such pitfalls gives an increased insight into the principles of reasoning in general and thereby we can avoid stumbling upon them. Finally, a study of this discipline empowers the student with techniques and methods for testing the correctness of many different kinds of reasoning, and when errors are detected, they are removed at once. Again, problem with this definition is that whatever may be its merit, it is also subjective because reasoning depends upon the person who reasons. If there is no one who reasons, then there is no reasoning at all. Therefore this definition also does not take us far. As an alternative, logic was defined as the science of inference by some logicians. Though this definition is better than the older definitions, even this definition is not free from defect completely. Inference is a special form of mental activity. Its subjective nature becomes obvious when we notice that if there is some one who infers, then there is inference; not otherwise. However, very shortly we notice that inference is not banished altogether from the domain of logic and that it has a definite role to play in the development of logic. If so what is an acceptable definition of logic? Logic concerns with distinction between good argument and bad argument. This itself constitutes the definition or essence of logic. An argument always points to a certain relation between two sets of statements or propositions. One set is called premise or premises and another is called conclusion. If the conclusion follows from the premises, then the argument is said to be good; otherwise bad. How do we know whether the conclusion follows from the premises or not? As in the case of games here also total adherence to rules makes an argument good. Even if one rule is violated the argument turns out to be bad. It only means that conclusion follows from the premises only when all rules are scrupulously followed. At this stage, we introduce a technical word. We say that the premises imply the conclusion if the same follows from the given premises. Therefore implication is the desired relation between the premises and the conclusion. Implication is not something which is brought from outside. It is latent in the premises only. It is left to the intellect of human being to discover or to extract what is latent. Implication is objective and, therefore, man-independent because if it exists, it exists independent of any thinking mind. No amount of effort on the part of thinking minds can impose implication when it does not exist. It can only be discovered, but cannot be created. The process of discovering what is latent is known as inference. Logic is not concerned with the process as such, but with the end product of process, i.e., presence or absence of implication. This will bring us to the crucial distinction to be made. Inference can be valid or invalid. If inference has its basis in implication, then it is valid. On the other hand, if it does not enjoy the support of implication, then it is invalid. However, there is nothing like valid or invalid implication. Either there is implication or there is no implication. That is all. Secondly, statements imply; they do not infer. On the contrary, humans infer; they do not imply. Therefore any error lies only in human activity. No error can be discerned in the relation between 4 statements. In the third place, implication without inference (valid) is possible, but valid inference without implication is neither possible nor plausible. This sharp distinction has its tell-tale impact. Contrary to inference which is man-dependent implication is man-independent. Suppose that logic is defined as a study of inference. Then it becomes subjective. If I infer then only there is logic; otherwise not. On the contrary, if implication replaces inference, then logic becomes man-independent and hence objective. Rivalry between subjective and objective elements now surfaces. If knowledge is to be viewed as objective, then logic, automatically, ought to remain objective. Therefore implication replaces inference when we are concerned with the subject matter of logic. Though inference loses its place in this scheme, philosophers like Russell continued to use 'inference' only. Later we will learn that we have only rules of 'inference' but not rules of implication. The point to be noted is that in all these cases inference, paradoxically, means implication only. It is very important that this point is borne in our mind throughout our study of logic.

1.3 TWO TYPES OF LOGIC: FORMAL AND MATERIAL

Traditionally logic has been classified into two types 1) Formal and 2) Material logic. Formal logic is otherwise known as deductive logic and material logic as inductive logic. Formal logic is concerned with the form or structure of argument whereas material logic is concerned with the matter or content of argument. When matter is irrelevant, material truth also is irrelevant. What matters in deductive logic is formal truth. By formal truth we mean logical relation between the premises and the conclusion. It is possible to know this kind of truth without knowing the content of the argument. In this case, it is sufficient if the argument follows the rules of the game. This whole explanation can be put in a nut-shell in this manner. An argument consisting of only true propositions can very well be invalid whereas an argument consisting of only false propositions can very well be valid. It also means that in our study of deductive logic it is possible to know whether an argument is valid or not without knowing the contents of the argument (and many times this is what precisely happens) provided we are in a position to decide whether the argument has followed all the rules are not. However, the case of material logic is different. In this case it is possible to judge the truth or falsity of the conclusion only when we know what the argument is all about. What is more important than the previous statement is the controversy surrounding the relevance of rules. The burning question is whether there is anything like rule or rules governing the structure of inductive argument (for more details see, 1.4 of block 2). Suppose that there are no rules regulating inductive arguments as maintained by some philosophers. Then inductive arguments are neither valid nor invalid. If so, what is its status? A question like this is easier asked than answered. Attempts to answer this question occupy a good deal of discussions on inductive logic.

This reference provides some of the basic points made in Chapter One. But it doesn't include everything of importance! Please spend the time working through all the tutorials. Often details for working homework problems -- the only good preparation for exams -- is available in the tutorials. Even if you can do this weeks homework without doing them all, there may be material in later units that will be very hard without a clear understanding of all that going on in the tutorials.

In this unit we introduce the general idea of logic, the study of correct reasoning. We start with the general notion of an argument and develop the concepts needed for argument analysis and evaluation.

1. Arguments and Form: Basics

Let's keep our first example in mind about Chris as we go over the basic ideas of logic. The primary notion is that of an **argument**.

An **argument** is a collection of statements including some (the **premises**) that are given as reasons for another (a **conclusion**).

Here's the familiar example ...
Chris will get an "A" or a "B" in logic class.

Chris (it turns out) does not get an "A".

So, Chris will get a "B".

...and here are some of the basic ideas:

- Reasoning is often expressed by an argument.

- An argument's premises are its statements of evidence.

- The premises of an argument are meant to *support* its conclusion, that is, to give reasons to believe the conclusion is true.

- The form of an argument is the way its language is structured.

- An informal argument is meant to conclude with a kind of best guess...it's conclusion should be an aspect of the best interpretation of the data or evidence.

Usually ***formal logic*** can also be called ***deductive logic*** because the form of thinking allows one to deduce it's conclusion from its premises (as in the Chris process of elimination example argument described just above). ***Informal logic*** is usually called ***inductive logic***. Reasoning based on informal, inductive logic moves from statements of evidence (the premises) to a conclusion that extrapolates from, amplifies, or generalizes the evidence.

The process of elimination argument form we've been seeing will henceforth be called DS. The Chris "A" or "B" argument is an example:
Chris will get an "A" or a "B" in logic class.

Chris (it turns out) does not get an "A".

So, Chris will get a "B".

That is to say:

Any argument with the form: "Either A or B, but not-A, so B" is called **DS**.
Also, suppose Chris does *better*, he doesn't get a 'B', then still assuming that Chris will get an 'A' or a 'B', it follows that Chris will receive an 'A'. This is also DS.

Thus, there is a second version of the form **DS**: "Either A or B, but not-B, so A". [Note the slight difference: the second premise is "not-B" rather than "not-A".

Here's another example.

If Chris gets an A, *then* he will be very happy. And (as it turns out) he *does* get an 'A'. So, it follows that Chris will be very happy.

This is an argument of a form we'll name as follows.
Any argument of the form "If A then B, and A so B" is called **MP** or "Modus Ponens".

We'll get lots of practice with this. So spend a moment to make some sense of it and then move on...

Notes

If Chris gets an A, *then* he'll be very happy. But he turns out to be unhappy. So, it follows that Chris did *not* get an 'A'.

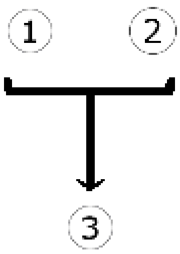
This is a different form:

Any argument of the form "If A then B, but not-B, so not-A" is called **MT** or "Modus Tolens".

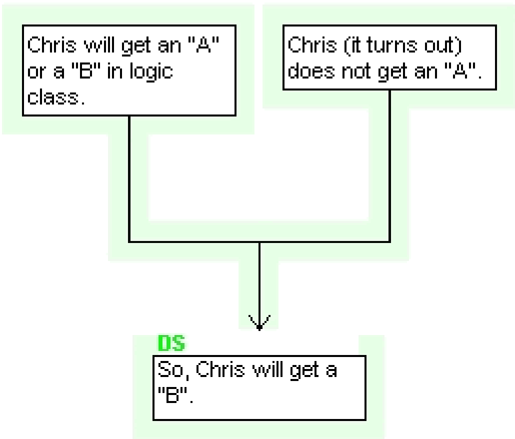
These forms will turn out to have great significance. In this chapter, we will just begin to see why that is so.

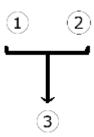
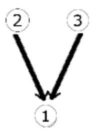
"Picturing" Arguments

We will often need to display arguments in a way that makes its premises- conclusion structure as clear as possible. The first method we've seen for doing this is the **Tree Diagram** (from tutorials 1 and 4). Here is an example.



or



Tree diagrams are especially useful for complex argument stuctures with more than one premise. The *merging* arrow used in this diagram,  , indicate that the premises, (1) and (2), work together or "collaborate" to support the conclusion. *Simple* arrows, as in this diagram,  , indicate that the premises each supports the conclusion independently of the other.

For simpler arguments, it's often better to give **Standard Diagrams** (see tutorial 2):

Chris will get an "A" or a "B" in logic class.

He does not get an "A".

Chris will get a "B".

To give a standard diagram, we write *the premises first*, draw a line, *then write the conclusion*. Here's another examle.

Chris has done well at college.

He has high LSAT scores.

Chris will likely be admitted to law school.

2. Determining Arguments and their Components

Distinguishing Arguments From Non-Arguments

We need to keep in mind that there are many types of thinking in language that do not give arguments.

Here are five types of passage that you'll need to be able to disentangle.

- An argument...gives reasons meant as evidence to support a conclusion, to show it true.
- An explanation...gives reasons but is meant to
- show *why* something is the case...if you notice that the lights go out, and ask why, then I tell you it’s a power failure. I’m not trying to convince you that the lights are out. The explanation gives an account showing how and why this happened.
- A conditional statement...if P then Q...as we’ve seen, these statements can be either premises or conclusions but they are not whole arguments. Think about MT or MP.
- A Report...just describes. E.g., I may describe all the reasons I love MX. It may be relevant to some conclusion you have in mind (“we should all go to MX this spring?); but if the conclusion is not drawn the passage is just a report.

- An illustraton...gives an example. “Philosophers are often picky about language. Thus Halpin doesn’t like me to say ‘I know it’ when I should say ‘I think so’. “Thus” means “So, for example” rather than “So, here’s my reason”.

Premise And Conclusion Indicators

Suppose we do have an argument.

All living beings deserve respect because life is sacred and the sacred deserves the greatest respect.

Now, how do we distinguish premises from conclusions or from other sentences which are not parts of an argument? For instance, in the example above arguing that life is sacred, how do we tell the conclusion from the premises? It's not always easy, but in this example there is a good hint. The word "**because**" is a premise indicator; it signifies that a premise *follows*. There are a number of roughly equivalent words or phrases in English; we'll call them all premise indicators. Several of the most common can be found in this table.

Premise Indicators:	
because	for the reason that
since	for the following reason
for	on account of

Now, on the other hand, we sometimes write things like

I've worked hard all morning so I deserve a good break this afternoon.

Here, the word "so" indicates that the conclusion is about to be given. We call it a "**conclusion indicator**". Again, there are many ways of indicating a conclusion. A number of them are given in the following table.

Conclusion Indicators:		
so	therefore	as a result
thus	it follows that	consequently
hence	in conclusion	so one can conclude

3. Distinguishing and Judging Arguments: Validity and Soundness

One of the main points of logic is to be able to distinguish good reasoning from bad. There are two main parts to this process: (1) the judgment of the force or support of premises for conclusion and (2) the judgment of the correctness of the premises. The strongest sort of force or support is associated with valid arguments. The idea is that so long as the premises are assumed to be true, the conclusion is inescapable. We make this a bit more precise in the following terms:

An argument is **valid** just in case it is not possible that its conclusion be false while its premises are all true.

An argument is **invalid** if and only if it is not valid.

So the definition of validity (the property of being valid) has to do with (1). Our second definition combines judgments (1) and (2):
An argument is **sound** if and only if it is both (a) valid and (b) has only true premises.

An argument is **unsound** if and only if it is not sound.
But it can be a bit disconcerting to decide on soundness (the property of being sound)! That takes us rather far from the province of logic. So, it's good to point out that an argument's soundness is something that we won't often be able to decide as a matter of logic. When you are examined on soundness, you can expect matters that are fairly uncontroversial.

Think about the following argument. It's very uncontroversial and really rather uninteresting. But that makes it easier to judge.

All whales are mammals.

The animal who played Free Willy is a whale.

The animal who played Free Willy is a mammal.

Notice first that this argument is valid. Even if you don't know anything about whales or Free Willy, it's clear that the conclusion is inescapable *given* that the two premises (the statements above the line) are true. Second, the premises *are* true. So, the argument meets the two conditions required for it to be sound.

Now, consider another argument.

All whales live in the Southern Hemisphere.

Shamu (of San Diego, CA) is a whale.

Shamu lives in the Southern Hemisphere.

This argument too is valid. How can you tell? A test is to imagine the premises being true. Here you might have to imagine herding all the whales south of the equator! But imagine it anyway. Then notice that you are automatically imagining the conclusion being true as well. It's impossible for the conclusion to be false while the premises too are true. So, the argument is valid. But, of course, it's not sound. It has a false premise -- imagining that all whales live south of the equator does not make it so.

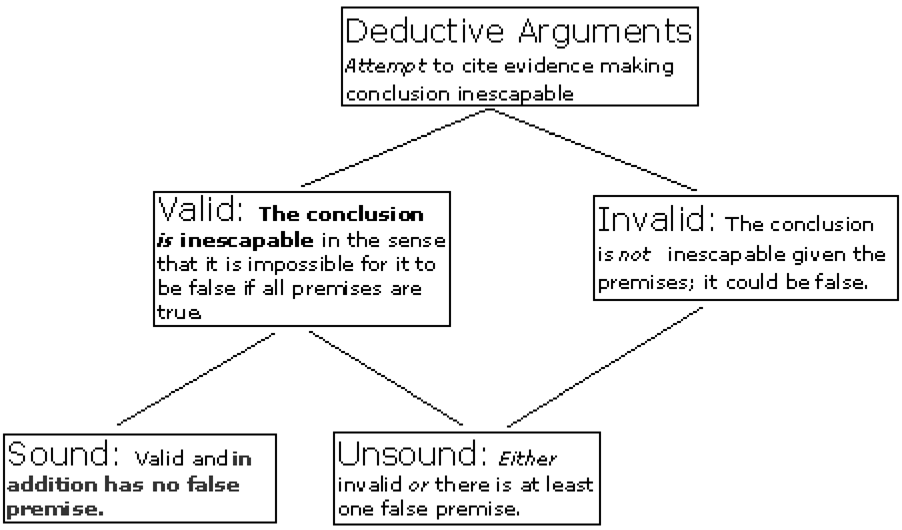
It's worth noting that when we are concerned with validity, actual truth or falsity of statements need not matter. Instead, validity is only concerned with what happens IF premises are true.

Only when we are concerned with soundness (or cogency in inductive logic) do we need to think about whether the premises and conclusion are in fact true or false.

Now, not all arguments are meant to be valid or sound. We can only give valid and sound arguments when we have the most forceful premises. When we do argue in this way, the reasoning is **deductive**; we'll say the study of such reasoning is "deductive logic".

An argument is **deductive** if and only if its premises are *intended* to lead to the conclusion in a valid way.

Note the word "intended" that is part of this definition. Whether or not an argument is deductive depends on how it is meant. Often we intend to give a valid argument but fail. (Didn't you ever give a "proof" in geometry class that was meant to validly imply some theorem, only to find you were wrong?) In any case, an argument may count as deductive even when it is not valid; judging an argument as deductive is a matter of interpretation not just logic.



4. Distinguishing and Judging Arguments: Inductive Reasoning

Frequently we need to give arguments even when our evidence only makes a conclusion *likely*, but not inescapable. Then our thinking is often called "inductive". For example,

I have surveyed hundreds of students here at ITU and found that less than 10% say they are happy with the new course fees. My sample was selected at random. So, I conclude with confidence that the vast majority of ITU students do not find the course fees acceptable.

Here, the argument's author is clearly claiming that the evidence cited makes the conclusion likely to be true but not a certainty (surveys sometimes do go badly awry, for instance when the participants have some reason to lie.) So, this argument is a clear case of an inductive argument.

An argument is **inductive** if and only if its premises are *intended* to lead to its conclusion with high probability.

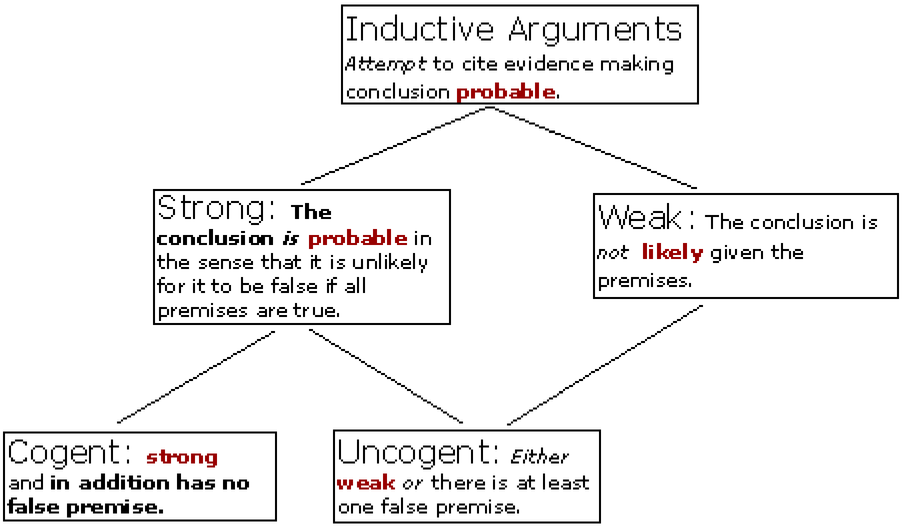
We do *not* say that an inductive argument is valid when it succeeds at supporting its premises as intended. This because an inductive argument does not intend to be valid, does not intend that its conclusion is inescapable. Rather, an inductive argument whose premises do support its conclusion as intended (i.e., they make the conclusion likely) is called "inductively strong":

An argument is **inductively strong** if and only if its conclusion is *highly probable* to be true given its premises.

Inductive strength is a counterpart to validity: by definition, deductive arguments are intended to be valid, inductive arguments are intended to be inductively strong. Of course, people often give arguments falling short of what was intended. That's why we have logic classes! But the point is that "valid" and "inductively strong" play similar roles for deductive and inductive arguments respectively: they support their conclusions as intended.

Finally, we need to define a counterpart to "sound" for inductive arguments. Remember, that an argument is sound if and only if it's both valid and has all and only true premises. For an inductive argument we just substitute "inductively strong" for "valid" to get the notion of cogency:

An argument is **cogent** if and only if it is both inductively strong and all its premises are true.



1.4 LOGIC: SCIENCE OR ART?

Features and problems of logic

Three areas of general concern are the following.

Logical semantics

For the purpose of clarifying logical truth and hence the concept of logic itself, a tool that has turned out to be more important than the idea of logical form is logical semantics, sometimes also known as model theory. By this is meant a study of the relationships of linguistic expressions to those structures in which they may be interpreted and of which they can then convey information. The crucial idea in this theory is that of truth (absolutely or with respect to an interpretation). It was first analyzed in logical semantics around 1930 by the Polish-American logician Alfred Tarski. In its different variants, logical semantics is the central area in the philosophy of logic. It enables the logician to characterize the notion of logical truth irrespective of the supply of nonlogical constants that happen to be available to be substituted for variables, although this supply had to be used in the characterization that turned on the idea of logical form. It also enables him to identify logically true sentences with those that are true in every interpretation (in "every possible world").

The ideas on which logical semantics is based are not unproblematic, however. For one thing, a semantical approach presupposes that the language in question can be viewed "from the outside"; i.e., considered as a calculus that can be variously interpreted and not as the all-encompassing medium in which all communication takes place (logic as calculus versus logic as language).

Furthermore, in most of the usual logical semantics the very relations that connect language with reality are left unanalyzed and static. Ludwig Wittgenstein, an Austrian-born philosopher, discussed informally the "language-games"—or rule-governed activities connecting a language with the world—that are supposed to give the expressions of language their meanings; but these games have scarcely been related to any systematic logical theory. Only a few other attempts to study the dynamics of the representative relationships between language and reality have been made. The simplest of these suggestions is perhaps that the semantics of first-order logic should be considered in terms of certain games (in the precise sense of game theory) that are, roughly speaking, attempts to verify a given first-order sentence. The truth of the sentence would then mean the existence of a winning strategy in such a game.

Limitations of logic

Many philosophers are distinctly uneasy about the wider sense of logic. Some of their apprehensions, voiced with special eloquence by a contemporary Harvard University logician, Willard Van Quine, are based on the claim that relations of synonymy cannot be fully determined by empirical means. Other apprehensions have to do with the fact that most extensions of first-order logic do not admit of a complete axiomatization; i.e., their truths cannot all be derived from any finite—or recursive (see below)—set of axioms. This fact was shown by the important "incompleteness" theorems proved in 1931 by Kurt Gödel, an Austrian (later, American) logician, and their various consequences and extensions. (Gödel showed that any consistent axiomatic theory that comprises a certain amount of elementary arithmetic is incapable of being completely axiomatized.) Higher-order logics are in this sense incomplete and so are all reasonably powerful systems of set theory. Although a semantical theory can be built for them, they can scarcely be characterized any longer as giving actual rules—in any case complete rules—for right reasoning or for valid argumentation. Because of this shortcoming, several traditional definitions of logic seem to be inapplicable to these parts of logical studies.

These apprehensions do not arise in the case of modal logic, which may be defined, in the narrow sense, as the study of logical necessity and possibility; for even quantified modal logic admits of a complete axiomatization. Other, related problems nevertheless arise in this area. It is tempting to try to interpret such a notion as logical necessity as a syntactical predicate; i.e., as a predicate the applicability of which depends only on the form of the sentence claimed to be necessary—rather like the applicability of formal rules of proof. It has been shown, however, by Richard Montague, an American logician, that this cannot be done for the usual systems of modal logic.

Logic and computability

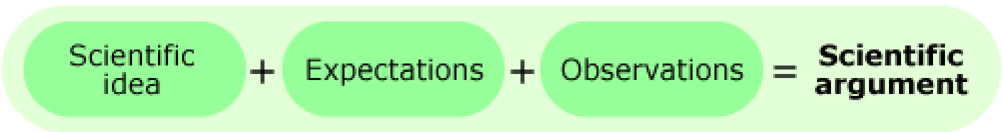
These findings of Gödel and Montague are closely related to the general study of computability, which is usually known as recursive function theory (see mathematics, foundations of: The crisis in foundations following 1900: Logicism, formalism, and the metamathematical method) and which is one of the most important branches of contemporary logic. In this part of logic, functions—or laws governing numerical or other precise one-to-one or many-to-one relationships—are studied with regard to the possibility of their being computed; i.e., of being effectively—or mechanically—calculable. Functions that can be so calculated are called recursive. Several different and historically independent attempts have been made to define the class of all recursive functions, and these have turned out to coincide with each other. The claim that recursive functions exhaust the class of all functions that are effectively calculable (in some intuitive informal sense) is known as Church's thesis (named after the American logician Alonzo Church).

One of the definitions of recursive functions is that they are computable by a kind of idealized automaton known as a Turing machine (named after Alan Mathison Turing, a British mathematician and logician). Recursive function theory may therefore be considered a theory of these idealized automata. The main idealization involved (as compared with actually realizable computers) is the availability of a potentially infinite tape.

The theory of computability prompts many philosophical questions, most of which have not so far been answered satisfactorily. It poses the question, for example, of the extent to which all thinking can be carried out mechanically. Since it quickly turns out that many functions employed in mathematics—including many in elementary number theory—are nonrecursive, one may wonder whether it follows that a mathematician's mind in thinking of such functions cannot be a mechanism and whether the possibly nonmechanical character of mathematical thinking may have consequences for the problems of determinism and free will. Further work is needed before definitive answers can be given to these important questions.

Questions have been raised on the issue whether logic is a science or an art or both. Let us stay for a while on this problem. In ancient times science just meant a systematic study of anything. But today the term science has developed into a discipline distinct from several other activities of mankind. Science has been defined as that branch of knowledge which aims at explanation of phenomena. Used in this technical sense, logic is no science at all. Does this mean that logic is an art? Art is concerned with doing something. Logic, if defined as an art, is so only in derivative sense. In order to decide whether or not logic is an art we have to consider the aim of logic. Is the aim of logic to give us knowledge about valid argument forms or to make us better thinkers? No one will deny that a study of logic results in improving our reasoning ability. But there is a restriction. Just like a moralist who may not himself be moral as a person, a logician may not be logical in his reasoning. We can say that the effect of such a study is the acquisition of knowledge regarding valid argument forms. It is not for logic to consider whether or not this knowledge is put into practice. In view of this feature we can say that logic is a science and not an art. It is a science not in the technical sense, but in a general sense.

Taken together, the expectations generated by a scientific idea and the actual observations relevant to those expectations form what we'll call a scientific *argument*. This is a bit like an argument in a court case — a logical description of what we think and why we think it. A scientific argument uses evidence to make a case for whether a scientific idea is accurate or inaccurate. For example, the idea that illness in new mothers can be caused by doctors' dirty hands generates the expectation that illness rates should go down when doctors are required to wash their hands before attending births. When this test was actually performed in the 1800s, the results matched the expectations, forming a strong scientific argument in support of the idea — and hand-washing!



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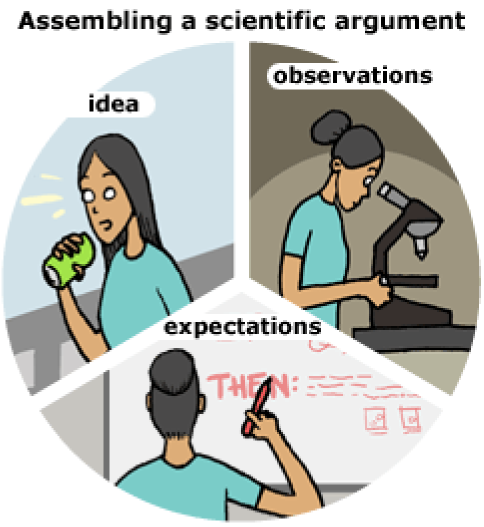

Though the elements of a scientific argument (scientific idea, expectations generated by the idea, and relevant observations) are always related in the same logical way, in terms of the process of science, those elements may be assembled in different orders. Sometimes the idea comes first and then scientists go looking for the observations that bear on it. Sometimes the observations are made first, and they suggest a particular idea. Sometimes the idea and the observations are already out there, and someone comes along later and figures out that the two might be related to one another.

Testing ideas with evidence may seem like plain old common sense — and at its core, it is! — but there are some subtleties to the process:

- **Ideas can be tested in many ways.** Some tests are relatively straightforward (e.g., raising 1000 fruit flies and counting how many have red eyes), but some require a lot of time (e.g., waiting for the next appearance of Halley's Comet), effort (e.g., painstakingly sorting through thousands of microfossils), and/or the development of specialized tools (like a particle accelerator). To explore further, jump to *Tactics for testing*.

- **Evidence can reflect on ideas in many different ways.** To explore further, jump to *Reviewing your test results*.

- **There are multiple lines of evidence and many criteria to consider in evaluating an idea.** To explore further, jump to *Competing ideas: A perfect fit for the evidence?* or *Competing ideas: Other considerations*.

- **All testing involves making some assumptions.** To explore further, jump to *Making assumptions*.

Despite these details, it's important to remember that, in the end, hypotheses and theories live and die by whether or not they work — in other words, whether they are useful in explaining data, generating expectations, providing satisfying explanations, inspiring research questions, answering questions, and solving problems. Science filters through many ideas and builds on those that *work*!

1.5 LOGIC: POSITIVE SCIENCE OR NORMATIVE SCIENCE?

Logic as a Positive Science is one of the major works of Italian Marxist philosopher Galvano Della Volpe. It was first published in 1950 as Logica come Scienza positiva. A second edition appeared in 1956 and according to translator, Jon Rothschild, Della Volpe was reportedly working on a third edition at the time of his death in 1968 which was never completed. The definitive, enlarged edition was published posthumously in 1969 under the slightly different title Logica Come Scienza Storica. Jon Rothschild translated the book into English for New Left Books (now Verso), and was first published by them as Logic as a Positive Science in 1980.

Granted that logic is a science, what type of science is it? Science has been classified into two types, viz.,

1) positive Science and

2) normative Science.

Positive science describes what the case is. Normative science, on the other hand, tells us what ought to be the case. Let us now examine whether logic is a positive science or a normative science. Some logicians consider logic to be a formal science and regard it as a normative science. Just like object thought is made up of form and matter. According to Latta & Macbeath 'the form of thought is the way in which we think of things, the matter of thought is the various particular objects we think of. A form is something which may remain uniform and unaltered, while the matter thrown into that form may change and vary. A normative science attempts to find out the nature of forms (standards) on which our judgments of value depend. Normative sciences have before them a standard with reference to which everything within the scope of science is to be judged. A normative science gives us judgments of value, i.e., it tells us what ought to be the case. Logic has an important normative aspect; but a norm or ideal in logic has a special meaning.

The main business of logic is to discover the general conditions on which the validity of inference depends. In our discussion of logic we try to force these conditions on the way of arguing. We do so because there are certain objective relations between statements. This means that statements must possess a certain structure and there must be certain objective relations between them if our inferences are to be valid. These structures of statements and their mutual relations are pure forms, which serve as norms in logic. Traditional logicians while considering logic to be a normative science meant that it is a science concerned with those principles which ought to be followed in order to attain the ideal of truth. Some other logicians consider logic to be a descriptive science or a positive science and not a normative science since it does not lay down any norm for thinking. Its nature is description as it aims at describing and classifying various types of arguments. In fact the classification of sciences into positive and normative cannot be applied to logic. Logic cannot be characterized either as positive or as normative science. If logic were a positive science, it would merely describe different argument forms. Logic however, does not do so. The logician aims to build a deductive system whose elements are logically true propositions (tautologies). These propositions are purely formal and hence have no reference to context. Similarly, logic cannot be considered normative. It does not search for principles on which value judgments depend. In fact, the starting point for logic is our ability to distinguish between valid and invalid arguments. The logician only makes explicit the principals involved in valid arguments. This discussion reveals that positive-normative distinction is not relevant in the context of logic.

Check Your Progress 1

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1) Bring out merits and demerits of various definitions of logic. ………………………………………………………………… ………………………………………………………………… ………………………………………………………………… ………………………………………………………………… ……………………

2) Is logic a Positive science or a Normative Science? Substantiate your position.
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1.6 LOGIC AND OTHER DISCIPLINES

Logic as a discipline has wide scope and this will be clear if we examine its relationship to various empirical and social sciences. Logic is closely associated with almost all disciplines. Some are very significant. Therefore a cursory reference to some of them is desirable. Logic and Epistemology: Epistemology is that branch of philosophy which deals with theories of knowledge. It investigates the structure, conditions, sources as well as limitations of human knowledge. Epistemology, though, is not a formal science like logic since it must deal subjective entities like belief it does make use of logic and its methods widely to form theories about it. In fact there is a subdivision within epistemology called epistemic logic which specifies the limits of logical norms applicable in epistemic situations. Though logic and epistemology are interrelated, we cannot attribute any genus – species relation between the two. Logic is the science of reflective thinking in so far as implications are concerned. The province of logic is confined to certain formal methodologies. Epistemology consists of a number of cognitive affairs which goes beyond logic. Similarly logic too extends outside the concerns of epistemology.

Logic and Metaphysics:

Traditionally, the subject matter of metaphysics is regarded as the nature of Being or Reality. Since Greek times metaphysics has been conceived as the mother of all knowledge and it is this subdivision of philosophy which examines every presupposition of various sciences. For instance, physics assumes the existence of matter, motion, force, time and space. It is metaphysics which takes upon itself this task of examining these presuppositions of various sciences. The basic assumption of logic is that thought gives knowledge. It is necessary that we enquire into this very presupposition. In this endeavour metaphysics comes to our aid. Again it is common to make a distinction between real and unreal. But inquiry into the basic nature of this distinction is not common. Metaphysics deals with this problem as well. Not only does it analyse the basis of all sciences, but also provides a criterion of reality. Logic in fact stands between metaphysics and science. Abstraction of the bases of the principles of science is done through logic which bridges the gap between metaphysics and sciences.

Logic and Psychology:

Tradition stipulates that logic and psychology are related on the basis of the assumption that thought is a common factor to them. However, a crucial point is missing in this correlation. The traditional approach is something like this. Psychology takes up the study of origin and evolution of thought process by examining the functions of animals, infants, abnormal persons, criminals etc. Its main concern is mind and thought is a mental process. Logic gets confined to the study of inferential thinking of only normal adult human beings. Again while logic attempts to abstract the forms in which human mind thinks, psychology studies the actual process of thinking. The forms of thinking which logic abstracts from our thought processes are not events in our mind and, therefore, are not of interest to psychologists. Being a formal science, logic looks upon those principles as regulative elements of reflective thinking. Psychology is concrete because its subject – matter is concrete, i.e., actual psychological events. Logic is abstract because its subject matter is abstract, i.e., forms of reflective thinking. Therefore in one sense they are related and in some other sense they are poles apart. Ironically, this is just a matter of history of psychology as well as logic because today psychology does not regard mind as the topic of its concern and thought is no longer reckoned as mental. It is at once transformed into a sort of neurological process though its subjective nature remains unaltered. Only in this sense psychology studies thought. And logic is anything but a study of thought. Hence it is really obsolete to relate logic and psychology. Therefore logic and psychology are distinct disciplines and have nothing in common. However, we can remark that there is something logical in psychology though there is nothing psychological in logical enterprise. This is so because no science can afford to be illogical and, admittedly, at least some sciences can progress without recourse to psychological elements.

A question is frequently asked; which one has wider application; logic or psychology? This is an unanswerable question. In one sense the province of psychology is wider than that of logic since the former studies the entire activities of the human mind. In a different sense logic is wider than psychology because the latter follows logical principles while dealing with its own subject matter. The two sciences are mutually complementary.

Logic and Language:

Language is only a means of expression, yet the nature of language affects logical thinking. Just as the success of an operation depends upon the quality of surgical instruments apart from the skill of the surgeon, the quality of the argument depends upon not merely the validity of the forms of thinking the agent resorts to, but on the language in which the arguments are expressed as well. Natural language performs multiple functions, like conveying information, evoking emotions, stimulating action, making reference and so on. The structure of natural language is so constituted that it enables the language to perform these diverse functions. However, language of logic needs to convey only information. Hence it calls forth the use of emotively neutral language. Logicians take extra care in using plain and non-sophisticated language so that they just convey information, which is either true or false. Logical statements pronounce that something is or is not the case. For instance, 'Atom has been split' is a factual statement which carries a definitive truth-value. Logic demands statements which convey exact information through a neutral use of language. Language is so subtle and complicated an instrument that we often lose sight of the multiplicity of its uses. But there is real danger in our tendency to over simplify. On the staggering variety of uses of language some order can be imposed by diving them into very general categories: the informative, the expressive and the directive. Among these three uses, logic is concerned only with the informative use of language. Many philosophers, however, have claimed that the structure of logic and language are identical. Therefore, a better understanding of logic depends upon the elimination of ambiguity and vagueness of language.

Logic and Physical Sciences:

Of late, science and scientific culture seem to shape human life. The goal of science is to study the natural events of various types and discover generalization regarding them. The generalisations are utilized to yield comprehensive theories about the working of nature. The procedure of science involves both observation of facts and reflective thinking. The principles of logic help science to analyse the observed facts and draw valid conclusions from them.

Logic and Mathematics:

Let us briefly dwell on the background before proceeding further. Though the beginnings of modern logic are found in the writings of Leibniz, it was not until the end of the nineteenth century that logic discovered new path of development. The shift in track was partly due to certain topics in mathematics which received the impetus and partly due to the discovery of paradoxes. These developments resulted not just in the overlapping of logic and mathematics, but at some point of time, it became 'extremely difficult to draw a non-arbitrary line between logic and mathematics'. In this section, only cursory reference can be made to important milestones which led to constant interplay between logic and mathematics. The ball was set rolling by George Boole when his work on 'The Mathematical Analysis of Logic' was published in 1847. The essence of his work was with his treatment of the logic of classes. This was followed by George Cantor's investigations on theory of sets. What made Cantor's work on theory of sets significant were his studies in analysis in general, and theory of trigonometric series in particular. However, the required breakthrough was provided by Gottlob Frege when he attempted to base mathematics on pure logic. In his own words, arithmetic is only a development of logic. Not only arithmetic became an extension of logic, but also due to the discoveries of non-Euclidean schools of geometry and certain paradoxes by Russell, Cantor and others at a later stage, mathematics itself was regarded as an extension of logic and this thesis came to be known as Frege-Russell thesis. This extension was described by Russell and Whitehead in their preface to the 'Principia Mathematica' as backward extension, thereby meaning extension to roots. G. Peano tried a different route to connect mathematics and logic. Instead of trying to secure a sound base in logic to mathematics, he analysed the methods of mathematics which were structurally similar to the calculus of logic and in this way he tried to link the two. None of these attempts aimed at 'mathematicising' logic so much as 'logicising' mathematics. Consequently, logic became the foundation of mathematics. Serious reservations against this theory came only from two quarters. Kronecker questioned the ideas of Cantor only to challenge the 'ostensible' essence of mathematics because he believed that Cantor's theory was not mathematics but sort of mysticism, a view partly endorsed by Cantor himself. Poincare was another philosopher who reacted in the same spirit to Zermelo's axiomatic set theory. Poincare' argued that the nature of natural number system is such that it is incapable of being reduced to logic.

He was more emphatically opposed to 'reducing' mathematical induction to logic. Surprisingly, he argued that mathematical concepts should be built up inductively by proceedings from 'particular' to 'general'. Perhaps he subscribed to the view that induction is not logic. A brief reference to of mathematical induction mentioned above is relevant. Mathematical induction is a misnomer because, in reality, there is no inductive element at all involved here, even though the principle proclaims that 'every natural number has a successor', i.e., if n is a natural number, then n+1 is also a natural number. This is the essence of mathematical induction. This theorem involves rigorous logical proof which is essentially deductive in nature with no semblance of inductive inference. It should be mentioned that Poincare' did not oppose mathematics following deductive model. Following a certain logical method is not the same as reducing a certain science to logic. Poincare' was only against making the latter.

If we go by the modern definition of mathematics as the science of formal proof or logical demonstration, then the relation between logic and mathematics becomes very intimate. Both logic and mathematics are formal sciences. They deal with relations between propositions which are independent of the content of the propositions. In arithmetic, for instance, we may use numbers to count anything. What we actually count makes no difference to counting. Thus two plus two will be four whether we add books, balls, tables or anything else. Since the relations with which logic and mathematics deal are independent of content these sciences are able to use symbols in place of words. Also, both logic and mathematics deal with relations which are applicable to actual as well as possible objects. Further, both logic and mathematics are deductive in character. They begin with certain axioms and deduce conclusions from them. Moreover, the method of both is a priori. Though both logical and mathematical operations may take place with reference to any empirical entity, knowledge of the principles of these disciplines is not gained by observation or sense experience. Such knowledge is called ‘a priori’, i.e., independent of experience.

Computers

It has already been indicated that recursive function theory is, in effect, the study of certain idealized automata (computers). It is, in fact, a matter of indifference whether this theory belongs to logic or to computer science. The idealized assumption of a potentially infinite computer tape, however, is not a trivial one: Turing machines typically need plenty of tape in their calculations. Hence the step from Turing machines to finite automata (which are not assumed to have access to an infinite tape) is an important one.

This limitation does not dissociate computer science from logic, however, for other parts of logic are also relevant to computer science and are constantly employed there. Propositional logic may be thought of as the “logic” of certain simple types of switching circuits. There are also close connections between automata theory and the logical and algebraic study of formal languages. An interesting topic on the borderline of logic and computer science is mechanical theorem proving, which derives some of its interest from being a clear-cut instance of the problems of artificial intelligence, especially of the problems of realizing various heuristic modes of thinking on computers. In theoretical discussions in this area, it is nevertheless not always understood how much textbook logic is basically trivial and where the distinctively nontrivial truths of logic (including first-order logic) lie.

Methodology of the empirical sciences

The quest for theoretical self-awareness in the empirical sciences has led to interest in methodological and foundational problems as well as to attempts to axiomatize different empirical theories. Moreover, general methodological problems, such as the nature of scientific explanations, have been discussed intensively among philosophers of science. In all of these endeavours, logic plays an important role.

By and large, there are here three different lines of thought.

(1) Often, only the simplest parts of logic—e.g., propositional logic—are appealed to (over and above the mere use of logical notation). Sometimes, claims regarding the usefulness of logic in the methodology of the empirical sciences are, in effect, restricted to such rudimentary applications. This restriction is misleading, however, for most of the interesting and promising connections between methodology and logic lie on a higher level, especially in the area of model theory. In econometrics, for instance, a special case of the logicians' problems of definability plays an important role under the title "identification problem." On a more general level, logicians have been able to clarify the concept of a model as it is used in the empirical sciences.

In addition to those employing simple logic, two other contrasting types of theorists can be distinguished:

(2) philosophers of science, who rely mostly on first-order formulations, and

(3) methodologists (e.g., Patrick Suppes, a U.S. philosopher and behavioral scientist), who want to use the full power of set theory and of the mathematics based on it. Both approaches have advantages. Usually realistic axiomatizations and other reconstructions of actual scientific theories are possible only in terms of set theoretical and other strong mathematical conceptualizations (theories conceived of as "set-theoretical predicates"). In spite of the oversimplification that first-order formulations often entail, however, they can yield theoretical insights because first-order logic (including its model theory) is mastered by logicians much more thoroughly than is set theory.

Many empirical sciences, especially the social sciences, use mathematical tools borrowed from probability theory and statistics, together with such outgrowths of these as decision theory, game theory, utility theory, and operations research. A modest but not uninteresting beginning in the study of their foundations has been made in modern inductive logic.

1.7 DEDUCTIVE AND INDUCTIVE LOGIC

Traditionally arguments have been classified into two types, viz., deductive and inductive arguments. Accordingly there are two divisions of logic, viz., deductive logic and inductive logic. Deductive logic has arguments that consist of premise or premises and a conclusion. In a deductive argument the conclusion necessarily follows from the premises. Furthermore, it is the characteristic of the deductive argument that if one accepts the premises one has to accept the conclusion. Such arguments are available in mathematics and geometry. Deductive argument is not concerned with truth and falsity, but it is concerned with validity and invalidity or consistency and inconsistency of arguments. Validity and invalidity are characteristics of arguments whereas truth and falsity are characteristics of propositions. There is another kind of argument which is known as inductive argument, the concern of inductive logic. According to one group of philosophers, inductive arguments are found in empirical sciences such as physics, sociology, psychology etc. This view is hotly debated. Law of causality constitutes the very basis of inductive arguments. Generalisations and predictions are the objectives of inductive arguments. Generalization is an important parameter of inductive logic. Therefore a brief description of what it means is necessary. Suppose that I observe ten crows which are black. Then I jump to the conclusion that all crows are black without observing other crows. Therefore the conclusion includes and goes beyond observation. Such conclusion is called generalization. Therefore mere acceptance of the truth of premises do not warrant acceptance of the truth of conclusion. The conclusion is rendered probable because it may be true or false. This is how probability enters the field of inductive logic.

Deductive reasoning is the process of reasoning from the general to the specific. Deductive reasoning is supported by deductive logic, for example:

From general propositions:

- All ravens are black birds.
- For every action, there is an opposite and equal reaction.
- To specific propositions such as:
- This bird is a raven, therefore it is black
- This rifle will recoil when it is fired.
- In contrast to inductive reasoning, the conclusions of deductive reasoning are as valid as the initial assumption. Deductive reasoning was first described by the ancient Greek philosophers such as Aristotle.

Inductive reasoning is the process of reasoning from the specific to the general. Inductive reasoning is supported by inductive logic, for example:

From specific propositions such as:

- This raven is a black bird.

- This rifle recoils when it is fired.
- To general propositions:
- All ravens are black birds.
- For every action, there is an equal and opposite reaction.
- In contrast to deductive reasoning, conclusions arrived at by inductive reasoning do not necessarily have the same validity as the initial assumptions.

Check Your Progress 2

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1) State the relation between logic and language.

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2) Distinguish between deductive and inductive arguments.

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1.8 LET US SUM UP

Humans are endowed by nature with powers of reasoning. Logic is the study of the use of those powers. In the study of logic we come to recognize our own native capacities, and practice helps us to sharpen them. The study of logic helps one to reason well by illuminating the principles of correct reasoning. Correct reasoning is useful wherever knowledge is sought. Whether in science, politics or in the conduct of our private lives, we use logic in reaching defensible conclusions. In formal study we aim to learn how to acquire reliable information and how to evaluate competing claims for truth. Various definitions of logic were discussed and also types. Questions regarding the status of logic as an academic discipline were addressed subsequently. Arguments for and against logic as a science/ art, and logic as a positive science/ normative science, were discussed. The relevance scope of logic was examined by looking into the relation logic has with various other branches of knowledge. At the close of the unit, deduction and induction, the two major types of logic have been introduced to the student.

1.9 KEY WORDS

Logos: Logos is an important term in philosophy, analytical psychology, rhetoric and religion. Heraclitus (ca. 535–475 BCE) established the term in Western philosophy as meaning both the source and fundamental order of the cosmos. The sophists used the term to mean discourse, and Aristotle applied the term to rational discourse. After Judaism came under Hellenistic influence, Philo adopted the term into Jewish philosophy. The Gospel of John identifies Jesus as the incarnation of the Logos, through which all things are made. The gospel further identifies the Logos as divine (theos).

Positive Science: In the humanities and social sciences, the term positive is used in a number of ways. One usage refers to analysis or theories which only attempt to describe how things are, as opposed to how they should be. In this sense, the opposite of positive is normative. An example for positive, as opposed to normative, could be economic analysis. Positive statements are also often referred to as descriptive statements.

1.10 QUESTIONS FOR REVIEW

1) Bring out merits and demerits of various definitions of logic.
2) Is logic a Positive science or a Normative Science? Substantiate your position.

3) State the relation between logic and language.
4) Distinguish between deductive and inductive arguments.

1.11 SUGGESTED READINGS AND REFERENCES

- Copi, Irving M. & Cohen, Carl. Introduction to Logic. New Delhi: Prentice Hall of India, 1997.
- Copi, Irving. M. Symbolic Logic. Delhi: Prentice Hall of India, 2005.
- Das, G. Logic: Deductive & Inductive. Delhi: King Books, 1684.

1.12 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress 1

1) Bring out the various definitions attributed to logic? Logos – literal meaning: word, thought – eventually logic acquired a technical meaning – definition: Study of methods and principles which we use to distinguish good (correct reasoning) from bad (incorrect) reasoning – also defined as science of the laws of thought – again as science of reasoning.

3) Logic: is it a positive science or normative science? Substantiate your position. Positive science: describes what is the case – Normative science: tells us what ought to be the case – Formal science is that which takes up the form of the subject content for study – Normative science follows the norms – gives judgments of value – some logicians characterize it as positive science as well for its nature is description. It aims at describing and classifying various types of reasoning.

Check Your Progress 2

Notes

1) What is the relation between logic and language? Language affects logic – Natural language is an inconvenient tool to operate logical functions – Natural language being endowed with potency to attend divergent functions cannot get confined to the single function of conveying information – hence it calls forth the use of emotively neutral language – three functions of language: informative, expressive and directive – of these only informative use is conducive to logic.

2) Distinguish between deductive and inductive logic Historically logic has been divided into two – deductive and inductive. In deductive logic an argument's conclusion necessarily follows from the premises. Such arguments are available in mathematics and geometry. In a deductive argument we are concerned with validity and invalidity. Inductive logic has arguments that are found in empirical and social sciences. Generalizations and predictions are the objectives of inductive arguments.

UNIT 2: CONCEPT AND TERM

STRUCTURE

2.0 OBJECTIVES

Logic is said to be the study of argument as expressed in language. Language in general is highly ambiguous. In any language words are often used in various senses. For example, sometimes 'thought' and knowledge' are used as synonymous terms. Therefore this unit aims at:

- To examining major entities in the language of logic like concepts, words and terms and thereby show that they carry lot of philosophical significance and at the same time carry different senses.

- To familiarizing the students with varieties of technical terms. Terms can be classified under different criteria. There are simple and composite terms, singular and general terms, collective and non-collective terms, absolute and relative terms, concrete and abstract terms, positive, negative and private terms and connotative and non-connotative terms. This unit undertakes a study of these various types of terms to procure a good undertaking of the language of logic with which the student is expected to be sufficiently acquainted.

2.1 INTRODUCTION

Language is the external expression of intention, thought etc. It is the means of communicating our ideas to other people. In logic, by language we mean only the verbal expression of our ideas, either spoken or written. The Greeks seem to have used the word 'logos' to denote ideas as well as speech. This clearly shows the close relation between language and thought. As Grumbley has rightly pointed out, thought and language are closely connected just like how principles of life and activities of a living organism are connected. Clear thinking and accurate language help each other. Further, it is language that gives thought a name and an abiding reality as a permanent possession. It is popularly said: 'Thought lives in language'. The multitudes of objects that we see around cannot be remembered unless certain names are endowed to the ideas of those objects. The nature of thought is such that it gets dissolved unless we put them in words. Language not only structures thought by codifying them, but also does the service of preserving them for future generations. It enables us to split complex ideas into atomic ones to analyse and thereby understand them. Hence the philosophers often comment that logic and language are the two sides of the same coin. In order to understand complex ideas we split them into simple words. Words like chastity, nationality, and religious are just a few examples to convince ourselves that words can stand representing condensed expression of complex thoughts pregnant with many ideas. In this unit an attempt is made to understand how words, concepts and terms play a decisive role in our study of logic.

Issues And Developments In The Philosophy Of Logic

In addition to the problems and findings already discussed, the following topics may be mentioned.

Meaning and truth

Since 1950, the concept of analytical truth (logical truth in the wider sense) has been subjected to sharp criticism, especially by Quine. The main objections turned around the nonempirical character of analytical truth (arising from meanings only) and of the concepts in terms of which it could be defined—such as synonymy, meaning, and logical necessity. The critics usually do not contest the claim that logicians can capture synonymies and meanings by starting from first-order logic and adding suitable further assumptions, though definitory identities do not always suffice for this purpose. The crucial criticism is that the empirical meaning of such further "meaning postulates" is not clear.

Logical semantics of modal concepts

In this respect, logicians' prospects have been enhanced by the development of a semantical theory of modal logic, both in the narrower sense of modal logic, which is restricted to logical necessity and logical possibility, and in the wider sense, in which all concepts that exhibit similar logical behaviour are included. This development, initiated between 1957 and 1959 largely by Stig Kanger of Sweden and Saul Kripke of the U.S., has opened the door to applications in the logical analysis of many philosophically central concepts, such as knowledge, belief, perception, and obligation. Attempts have been made to analyze from the viewpoint of logical semantics such philosophical topics as sense-datum theories, knowledge by acquaintance, the paradox of saying and disbelieving propounded by the British philosopher G.E. Moore, and the traditional distinction between statements de dicto ("from saying") and statements de re ("from the thing"). These developments also provide a framework in which many of those meaning relations can be codified that go beyond first-order logic, and may perhaps even afford suggestions as to what their empirical content might be.

Intensional logic

Especially in the hands of Montague, the logical semantics of modal notions has blossomed into a general theory of intensional logic; i.e., a theory of such notions as proposition, individual concept, and in general of all entities usually thought of as serving as the meanings of linguistic expressions. (Propositions are the meanings of sentences, individual concepts are those of singular terms, and so on.) A crucial role is here played by the notion of a possible world, which may be thought of as a variant of the logicians' older notion of model, now conceived of realistically as a serious alternative to the actual course of events in the world. In this analysis, for instance, propositions are functions that correlate possible worlds with truth-values. This correlation may be thought of as spelling out the older idea that to know the meaning of a sentence is to know under what circumstances (in which possible worlds) it would be true.

Logic and information

Even though none of the problems listed seems to affect the interest of logical semantics, its applications are often handicapped by the nature of many of its basic concepts. One may consider, for instance, the analysis of a proposition as a function that correlates possible worlds with truth-values. An arbitrary function of this sort can be thought of (as can functions in general) as an infinite class of pairs of correlated values of an independent variable and of the function, like the coordinate pairs (x, y) of points on a graph. Although propositions are supposed to be meanings of sentences, no one can grasp such an infinite class directly when understanding a sentence; he can do so only by means of some particular algorithm or recipe (as it were), for computing the function in question. Such particular algorithms come closer in some respects to what is actually needed in the theory of meaning than the meaning entities of the usual intensional logic.

This observation is connected with the fact that, in the usual logical semantics, no finer distinctions are utilized in semantical discussions than logical equivalence. Hence the transition from one sentence to another logically equivalent one is disregarded for the purposes of meaning concepts. This disregard would be justifiable if one of the most famous theses of Logical Positivists were true in a sufficiently strong sense, viz. that logical truths are really tautologies (such as It is either raining or not raining”) in every interesting objective sense of the word. Many philosophers have been dissatisfied with the stronger forms of this thesis, but only recently have attempts been made to spell out the precise sense in which logical and mathematical truths are informative and not tautologous.

2.2 CONCEPT, WORD AND TERMS

It is necessary to distinguish between word, concept and term. Concept means a general idea. There is difference between the two ideas represented by the terms ‘student’ and ‘a student’. The term ‘a student’ refers to a particular student in an indefinite manner and it is essentially singular in usage. The term ‘student’ is applied in general to all those who undertake studies. The common and essential attributes which are found in every particular individual of the class are thought of separately, and thus we get a concept. In brief, the concept stands for general ideas. When expressed in language concept becomes a term. Judgment is the process of relating two concepts. For instance, the two concepts ‘water’ and ‘cold’ may be related and the result is the judgement, ‘water is cold’. A judgment when expressed in language is called a proposition. Sometimes it is said that concepts are mental entities. They are not visible. Conception or simple apprehension is the function of human mind by which an idea of a concept is formed in the mind. It is a process of forming mental image of an object, e.g., you see an elephant and form an idea of the elephant in the mind. The formation of concepts involves the fallowing processes.

(1) Comparison: Different entities are compared with one another so that the attributes they share in common and those on which they differ can be specified. This process enables the agent to find essential attributes of the concept and distinguish them from what are merely accidental.

(2) Abstraction: The next step is to abstract the essential characteristics. This is purely an intellectual exercise.

(3) Generalization: The third step is to generalize the result of abstraction because obviously not all objects belonging to any given class are observed.

(4) Naming: The final step is to give a name to the generalized group of common attributes, so that it becomes easy to retain the idea of the concept in our mind. Regarding the nature of conception, there are three views prevalent in metaphysics, Realism, Conceptualism and Nominalism. According to realism there is a corresponding real substance to every concept. This view is attributed to ancient Greek thinkers like Plato. Conceptualism is the view according to which conceptions are not real things but only general ideas. Nominalism is the view according to which conception are merely general names, not general ideas.

What is a word? A word consists of a letter or combination of letters conveying determinate meaning. A word may consist of only one letter. e.g. a, I, or it may consist of more than one letter, e.g., an, man, horse, mortal etc. A name is a word or group of words which can become the subject or predicate of a proposition. Every word cannot be called name, e.g., ‘or’, ‘before, ‘if’ etc. If we say ‘Before has four legs’ it sounds stupid. Thus it is clear that all words can not become names while all names must be words. Hobbes defines name thus: “A name is a word taken at pleasure to serve for a mark which may raise in our mind a thought like some thought which we had before, and which being pronounced by either, may be to them a sign of what thought the speaker had before his mind”. Mill also speaks of two kinds of words: words which are not names (as described above) and words which are names. He calls the latter term. A term is a word, or a combination of words, which by itself is capable of being used as subject or predicate of a proposition. A proposition is a declarative statement which is either true or false but not both. A term is so called because it occurs at the boundaries of a proposition. In the proposition 'Gandhiji is the father of the nation', 'Gandhiji, and 'father of the nation' are terms because they occur at the boundaries of the proposition. Traditional logic speaks of two kinds of words, viz., subject and predicate. In the example quoted above 'Gandhiji' is the subject because the proposition says something about 'Gandhiji' and 'father of the nation' is the predicate because it says something about subject, i. e., 'Gandhiji'. It means that subject term is that about which something is said and the predicate term is what is said about subject term. Here 'is' is not a term because it is incapable of functioning either as a subject or as a predicate. Also, names become terms only if they are parts of a proposition as subject and predicate. Thus every word is not term though every term is a word or a combination of words. Again, names may have different meanings, but a term has only one definite meaning in a proposition. Outside the proposition a term loses its significance and is merely a name. For example, Balance means a weighing machine or whatever is left after expenditure. However, when we use it in the proposition 'Balance is a weighing machine', it carries only one meaning.

There are three kinds' words: Categorematic, syncategorematic and a-categorematic. A categoremtic word is one which can by itself be used as a term without the help of other words, e.g. pencil, clever, man, etc. In other words when a word is used independently either as a subject or a predicate in a statement it is called as categorematic word. Examples: Roses are red. (Here 'red' is used as a predicate.) Red is a color. (Here 'red' is used as a subject.) All nouns including proper nouns are categorematic words. Let us look at a negative example. Consider a statement; 'roses are very colourful', the word 'very' cannot be independently used. We will not write 'Roses are very', it makes no sense. Nor can we use it as a subject. Again, when we say some are red, we actually mean 'out of many objects only some objects are red. Although some appears to be a subject it really is not. We understand it by the context in which the statement is made. 'Maradona is a great football player'. Here Maradona is used independently the subject in the statement. In these examples the words 'roses', 'Maradona', 'colourful', etc., are categorematic words. A syncategorematic word is one which cannot be used independently as term, but which can only be used along with other words e.g., of, with, and, the, etc. It is a word that is used as part of a subject or a predicate, or a word that joins the subject and the predicate. Nouns, participles, pronouns, adjectives, etc., are categorematic words, while parts of speech like preposition, adverb, etc., are syncategorematic words. Let us look at a few examples. In the statement 'Some people are funny' 'some' is a part of the subject and so it is a syncategorematic word. The word 'are' joins the subject and predicate, and it is also a syncategorematic word. Let us look at a few more examples: consider the statement 'Computer is very fast'. Here 'is' and 'very' are parts of the predicate. They are syncategorematic words. In the statement 'the telephone is dead', the word 'the' and 'is' are syncategorematic. Again in the example 'the cat is under the chair', 'under', 'the', and 'is' are syncategorematic words. In fact all words other than nouns and emotive words like Ah! Ouch! Alas, are syncategorematic words. In brief, a categorematic word is one which can be used as a term by itself, without the support of other words and syncategorematic word is one which cannot be used as a term by itself, but can form term only when joined to one or more categorematic words. A-categorematic words merely express some exclamatory feelings or emotions. Examples: Ouch! Aha! Hurrah! Hymn! Alas! Oh! and similar such exclamations. The word acategorematic may be jaw- breaking, but the words in this classification are pretty easy to identify. It cannot become a term either singly or even when conjoined with other words such as interjection. This classification of words into three types have been determined by the presupposition that subject predicate form is the basic form and all other sentences or propositions have to be transformed into this form. At this stage we need to introduce two very important notions: denotation and connotation. In the first place terms are used to point out objects, to name and to identify them, e.g., the term 'man' refers to all human beings. When a term is applied to denote objects or show their number, it is said to be used in denotation. It means number, or the reference of a term. As for example, the term 'man' denotes several individuals like Plato Aristotle, Gandhi and others, and all men of past, present and future.

Denotation is also known as extension because it shows the extent or range of objects to which a term is applied. All the objects to which a term is extended constitute the extension of a term. Terms are used not only to denote objects but also to show their qualities. In other words, terms are used to describe the object. Every term has a meaning. It stands for certain qualities. The term ‘man’ for example, shows the qualities of man like, ‘animality’ ‘rationality’ etc. The function of suggesting qualities possessed by this object is known as connotation. Every term denotes certain objects and connotes certain qualities. Connotation is also known as intension because it refers to general qualities intended by a term. The extension of the term ‘college’ is all the various colleges, while its intention is the various qualities describing the term, namely educational institution giving higher education. When we say that connotation of a term consists of the attributes which describe, a question arises as to which attributes are meant by the connotation. There are three views regarding the exact meaning of connotation.

1) Objective view: according to this view connotation means all the attributes actually possessed by the object, all known and unknown. But since in logic we are not concerned with anything unknown, this view is not useful.

2) Subjective view: according to this view we must mention all the qualities known to us. But the subjective position will cause variations regarding the actual qualities of the entity and hence is not acceptable.

3) Logical view: according to this view connotation means only those common, essential qualities of the object on account of which the term is applied to the object. Non-essential qualities do not form part of the connotation even if they are common to the whole class.

Let us see a few examples classifying denotation and connotation. Examples:

TERM DENOTES CONNOTES Shoe All shoes a stiff outer covering of the foot Knife All knives an instrument for cutting Love No denotation Fondness, strong liking here are a few more examples of connotation and denotation of terms. Common noun: ‘dog’ Denotation: all the animals to which the term can be applied. Connotation: a wild or domestic animal of the same genus as the wolf.

Descriptive phrases always have a connotative meaning, but their denotation may be definite, indefinite or totally absent. A definite description can be replaced by a proper noun. Example: ‘The proximate island to the south of India’ can be replaced by ‘Sri Lanka’. An indefinite description can be replaced by a common noun. Examples: ‘Baby lion’ can be replaced by ‘cub’. ‘House for a dog’ can be replaced by ‘kennel’. Naturally if a term does not denote anything (like the term ‘love’) the question of replacing by common noun does not arise.

2.3 TERMS AS A NAME OF CLASS

Alternative logics

The natures of most of the so-called nonclassical logics can be understood against the background of what has here been said. Some of them are simply extensions of the “classical” first-order logic—e.g., modal logics and many versions of intensional logic. The so-called free logics are simply first-order (or modal) logics without existential presuppositions.

One of the most important nonclassical logics is intuitionistic logic, first formalized by the Dutch mathematician Arend Heyting in 1930. It has been shown that this logic can be interpreted in terms of the same kind of modal logic serving as a system of epistemic logic. In the light of its purpose to consider only the known, this isomorphism is suggestive. The avowed purpose of the intuitionist is to consider only what can actually be established constructively in logic and in mathematics—i.e., what can actually be known. Thus, he refuses to consider, for example, "Either A or not-A" as a logical truth, for it does not actually help one in knowing whether A or not-A is the case. This does not close, however, the philosophical problem about intuitionism. Special problems arise from intuitionists' rejection (in effect) of the nonepistemic aspects of logic, as illustrated by the fact that only a part of epistemic logic is needed in this translation of intuitionistic logic into epistemic logic.

Other new logics are obtained by modifying the rules of those games that are involved in the game-theoretical interpretation of first-order logic mentioned above. The logician may reject, for instance, the assumption that he possesses perfect information, an assumption that characterizes classical first-order logic. One may also try to restrict the strategy sets of the players—to recursive strategies, for example.

Among the oldest kinds of alternative logics are many-valued logics. In them, more truth values than the usual true and false are assumed. The idea seems very natural when considered in abstraction from the actual use of logic. But a philosophically satisfactory interpretation of many-valued logics is not equally straightforward. The interest in finite-valued logics and the applicability of them are sometimes exaggerated. The idea, however, of using the elements of an arbitrary Boolean algebra—a generalized calculus of classes—as abstract truth-values has provided a powerful tool for systematic logical theory.

If we view a term as a name of a class, the connotation of the term defines the essence of that class, while the denotation refers to the members of the class. Examples: jet, medicine, disease, sports all these words are terms or classes. Consider one example. Jet is a class of objects. A quality or qualities which make an object jet constitute connotation. All connotative qualities together determine a class. A class may have sub-classes. Example 1: disease – tropical disease, heart disease, skin disease While 'disease' is a class it has sub-classes like 'tropical disease', 'heart disease', 'skin disease' etc. These sub-classes may in turn have individual members or further sub-classes. For example, the sub-class, 'tropical disease', has as members 'malaria', 'typhoid', 'cholera' etc. 'Typhoid' is a sub-class having members like 'para-typhoid' etc. Example 2: Class – 'sports' 'Outdoor sports' is a sub-class of the class 'sports'. 'Cricket' is a member of the sub-class 'outdoor sports'. The relation of the member 'cricket' to the class 'sports' or to the sub-class 'outdoor sports' is class membership. The relation of the sub-class 'outdoor sports' to the class 'sports' is called class inclusion.

A class is used in object-oriented programming to describe one or more objects. It serves as a template for creating, or instantiating, specific objects within a program. While each object is created from a single class, one class can be used to instantiate multiple objects.

Several programming languages support classes, including Java, C++, Objective C, and PHP 5 and later. While the syntax of a class definition varies between programming languages, classes serve the same purpose in each language. All classes may contain variable definitions and methods, or subroutines that can be run by the corresponding object.

Below is an example of a basic Java class definition:

```
class Sample
{
  public static void main(String[] args)
  {
    String sampleText = "Hello world!";
    System.out.println(sampleText);
  }
}
```

The above class, named Sample, includes a single method named main. Within main, the variable sampleText is defined as "Hello world!" The main method invokes the System class from Java's built-in core library, which contains the out.println method. This method is used to print the sample text to the text output window.

Classes are a fundamental part of object-oriented programming. They allow variables and methods to be isolated to specific objects instead of being accessible by all parts of the program. This encapsulation of data protects each class from changes in other parts of the program. By using classes, developers can create structured programs with source code that can be easily modified.

NOTE: While classes are foundational in object-oriented programming, they serve as blueprints, rather than the building blocks of each program. This is because classes must be instantiated as objects in order to be used within a program. Constructors are typically used to create objects from classes, while destructors are used to free up resources used by objects that are no longer needed.

2.4 EXTENSION AND INTENSION

It is customary to use ‘extension’ instead of ‘denotation’ and ‘intension’ instead of ‘connotation’ when a term refers to a class. There is a reason why the words ‘extension’ and ‘intension’ are used while we deal with classes. A class may have sub-classes, sub-classes with further subclasses, and so on as we have seen. By ‘extension’ we would then mean the range of sub-classes or number of members within that class i.e., how extensive is the denotation of the term, or how wide the denotation of a term is? Intension means the sum of the qualities which describe a general name. The scope of application of the term is to all the members of the class, and this signifies extension. The qualities or properties of content or subject matter of the term signifies intension. Let us take the term ‘box’ as example. The extensional significance of ‘box’ consists of the objects to which this term may be applied. The intentional significance of the term ‘box’ is the sum of attributes which defines the class.

Intension and extension, in logic, correlative words that indicate the reference of a term or concept: “intension” indicates the internal content of a term or concept that constitutes its formal definition; and "extension" indicates its range of applicability by naming the particular objects that it denotes. For instance, the intension of "ship" as a substantive is "vehicle for conveyance on water," whereas its extension embraces such things as cargo ships, passenger ships, battleships, and sailing ships. The distinction between intension and extension is not the same as that between connotation and denotation.

We're now starting to consider how our minds represent the meanings of words. If someone asked you, "What's the meaning of the word pencil?" you'd probably be able to describe it — it's something you write with, it has graphite in it, it makes a mark on paper that can be erased, it's long and thin and doesn't weigh much. Or you might just hold up a pencil and say, "This is a pencil". Pointing to an example of something or describing the properties of something, are two pretty different ways of representing a word meaning, but both of them are useful.

One part of how our minds represent word meanings is by using words to refer to things in the world. The denotation of a word or a phrase is the set of things in the world that the word refers to. So one denotation for the word pencil is this pencil right here. All of these things are denotations for the word pencil. Another word for denotation is extension.

If we look at the phrase, the Prime Minister of Canada, the denotation or extension of that phrase right now in 2017 is Justin Trudeau. So does it make sense to say that Trudeau is the meaning of that phrase the Prime Minister of Canada? Well, only partly: in a couple of years, that phrase might refer to someone else, but that doesn't mean that its entire meaning would have changed. And in fact, several other phrases, like, the eldest son of former Prime Minister Pierre Trudeau, and the husband of Sophie Grégoire Trudeau, and the curly-haired leader of the Liberal Party all have Justin Trudeau as their current extension, but that doesn't mean that all those phrases mean the same thing, does it? Along the same lines, the phrase the President of Canada doesn't refer to anything at all in the world, because Canada doesn't have a president, so the phrase has no denotation, but it still has meaning. Clearly, denotation or extension is an important element of word meaning, but it's not the entire meaning.

We could say that each of these images is one extension for the word bird, but in addition to these particular examples from the bird category, we also have in our minds some list of attributes that a thing needs to have for us to label it as a bird. That mental definition is called our intension. So think for a moment: what is your intension for the word bird? Probably something like a creature with feathers, wings, claws, a beak, it lays eggs, it can fly. If you see something in the world that you want to label, your mental grammar uses the intension to decide whether that thing in the word is an extension of the label, to decide if it's a member of the category. The next unit will look more closely at how our intensions might be organized in our minds.

One other important element to the meaning of a word is its connotation: the mental associations we have with the word, some of which arise from the kinds of other words it tends to co-occur with. A word's connotations will vary from person to person and across cultures, but when we share a mental grammar, we often share many connotations for words. Look at these example sentences:

- Dennis is cheap and stingy.

- Dennis is frugal and thrifty.

Both sentences are talking about someone who doesn't like to spend much money, but they have quite different connotations. Calling Dennis cheap and stingy suggests that you think it's kind of rude or unfriendly that he doesn't spend much money. But calling him frugal and thrifty suggests that it's honourable or virtuous not to spend very much. Try to think of some other pairs of words that have similar meanings but different connotations.

To sum up, our mental definition of a word is an intension, and the particular things in the world that a word can refer to are the extension or denotation of a word. Most words also have connotations as part of their meaning; these are the feelings or associations that arise from how and where we use the word.

In logic and mathematics, an intensional definition gives the meaning of a term by specifying necessary and sufficient conditions for when the term should be used. In the case of nouns, this is equivalent to specifying the properties that an object needs to have in order to be counted as a referent of the term.

For example, an intensional definition of the word "bachelor" is "unmarried man". This definition is valid because being an unmarried man is both a necessary condition and a sufficient condition for being a bachelor: it is necessary because one cannot be a bachelor without being an unmarried man, and it is sufficient because any unmarried man is a bachelor.

This is the opposite approach to the extensional definition, which defines by listing everything that falls under that definition – an extensional definition of bachelor would be a listing of all the unmarried men in the world.

As becomes clear, intensional definitions are best used when something has a clearly defined set of properties, and they work well for terms that have too many referents to list in an extensional definition. It is impossible to give an extensional definition for a term with an infinite set of referents, but an intensional one can often be stated concisely – there are infinitely many even numbers, impossible to list, but the term "even numbers" can be defined easily by saying that even numbers are integer multiples of two.

Definition by genus and difference, in which something is defined by first stating the broad category it belongs to and then distinguished by specific properties, is a type of intensional definition. As the name might suggest, this is the type of definition used in Linnaean taxonomy to categorize living things, but is by no means restricted to biology. Suppose one defines a miniskirt as "a skirt with a hemline above the knee". It has been assigned to a genus, or larger class of items: it is a type of skirt. Then, we've described the differentia, the specific properties that make it its own sub-type: it has a hemline above the knee.

Intensional definition also applies to rules or sets of axioms that define a set by describing a procedure for generating all of its members. For example, an intensional definition of square number can be "any number that can be expressed as some integer multiplied by itself". The rule—"take an integer and multiply it by itself"—always generates members of the set of square numbers, no matter which integer one chooses, and for any square number, there is an integer that was multiplied by itself to get it.

Similarly, an intensional definition of a game, such as chess, would be the rules of the game; any game played by those rules must be a game of chess, and any game properly called a game of chess must have been played by those rules.

An extensional definition of a concept or term formulates its meaning by specifying its extension, that is, every object that falls under the definition of the concept or term in question.

For example, an extensional definition of the term "nation of the world" might be given by listing all of the nations of the world, or by giving some other means of recognizing the members of the corresponding class. An explicit listing of the extension, which is only possible for finite sets and only practical for relatively small sets, is a type of enumerative definition.

Extensional definitions are used when listing examples would give more applicable information than other types of definition, and where listing

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the members of a set tells the questioner enough about the nature of that set.

This is similar to an ostensive definition, in which one or more members of a set (but not necessarily all) are pointed out as examples. The opposite approach is the intensional definition, which defines by listing properties that a thing must have in order to be part of the set captured by the definition.

2.5 INVERSE VARIATION

As the extension increases (covering more sub-classes), the intension decreases and if extension decreases, intension increases. The same relation can be stated in this way also. If intension increases, extension decreases and if intension decreases, extension increases. An example will clarify the relation. Let us employ hypothetical numbers and apply general knowledge. This is enough to understand the nature of relation.

Denotation	3 billion	1.1 billion	200 million	60 million
Terms	Asians	Indians	South Indians	Kannadigas
Intension	2	3	4	5

A person to be called an Asian must satisfy two requirements; 1) he or she must be a human being 2) that person must be a permanent resident within the geographical boundaries of Asia. Therefore the connotation of 'Asian' is 2. And the population of Asia is approximately 3 billion. Therefore the denotation of the term 'Asians' is 3 billion. 'Indians' constitute a subclass of Asians. Therefore the population of India must be naturally less than that of Asia. An Indian is not only a human living in Asia, but also possesses an additional connotation. He must be a bona fide citizen of India. Therefore the connotation of Indians is one more than that of 'Asians'. The student is advised to try to grasp the rest. It is easy to notice that in this arrangement as denotation decreases connotation increases. If we reverse the arrangement, decrease in intension is accompanied by increase in denotation. The student is advised to experiment to satisfy himself or herself of the truth of this relation. applicable to null classes.

When two variables change in inverse proportion it is called as indirect variation. In indirect variation one variable is constant times inverse of other. If one variable increases other will decrease, if one decrease other will also increase. This means that the variables change in a same ratio but inversely.

General equation for an inverse variation is Y = K1x. Or XY = K which is constant. So the product of two variables is a constant for inverse variation.

A variable quantity A is said to vary inversely as another variable quantity B, when A varies as the reciprocal of B i.e., when A varies as 1B

Thus, if A varies inversely as B, we write A ∝ 1B or, A = m · 1B or, AB = m where'm (≠ 0) is the constant of variation. Hence, if one variable varies inversely as another, then the product of the corresponding values of the variables is constant.

Conversely, if AB = k where A and B are variables and k is a constant, then A ∝ 1B1B.
Hence, if the product of the corresponding values of two variables is constant, then one quantity varies inversely as another.

Again, if A varies inversely as & then AB = constant; but AB = constant implies that when A increases in a given ratio, B decreases in the same ratio and vice -- versa. Thus, if two variables are so related that an increase (or decrease) in the value of one variable in a certain ratio corresponds to a decrease (or increase) in the same ratio in the value of the other variable then one variable varies inversely as another.

The relationship between variables is just the opposite of direct variation. When a car runs faster it takes less time to cover a distance. If speed of a car increase, it takes less time to cover a distance and vice versa. Here the distance is constant. It can be expressed as a inverse variation equation.

T = SVSV where T is time, S is the distance and V is the speed of the car.

Here S is the constant; T and V are variables which vary inversely.

a m.	20	16	40
b m.	8	10	4

Now we will solve some problems on direct variation:

1. If P varies indirectly as Q and the value of P is 4 and Q is 3, what is the equation that describes this indirect variation of P and Q?

Solution:

As P varies indirectly with Q, product of P and Q is constant for any value of P and Q.

So constant K = PQ = 4 × 3 = 12

So the equation that describes the direct variation of P and Q is P = 12Q12Q.

2. If a car runs at a speed of 40 kmph and takes 3 hrs to run a distance, what time it will take to run at a speed of 60 km?

Solution:

If T is the time taken to cover the distance and S is the distance and V is the speed of the car, the indirect variation equation is S = VT where S is constant and V and T are variables.

For the case given in the problem the distance that car covers is

S = VT = 40 × 3 = 120 km.

So at a speed of the car is 60 kmph and it will take

S = VT or 120 = 60 × T

T = 2 hrs.

3. In X is in indirect variation with square of Y and when X is 4, Y is 3. What is the value of X when Y is 6?

Solution:`

From the given problem indirect variation equation can be expressed as

X = KY2KY2 or K= XY^2

For the given case

K = 4 x 3^2 =36.

So when Y is 6,

XY^2 =36

or, X = 36Y236Y2

= 36623662

= 36363636

= 1

So the value of X is 1.

4. If A takes 5 days to complete a task when he works for 8 hrs a day, how many days he will take to complete the task if he works 5 hrs a day?

Solution:

If D is number of days, H is hrs of work in a day for A and T is the total time A takes to complete the task, from the given condition inverse variation equation is

D = THTH

or, T = HD, where T is constant.

T = 8 × 5=40.

So to complete the task A takes 40 hrs.

If he works 5 hrs a day

D = THTH = 405405 = 8.

So A will take 8 days.

Check Your Progress 1

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1) Distinguish between word, concept and term.

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2) Explain different classification of words.

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2.6 CLASSIFICATION OF TERMS

Terms are classified as simple and composite; Singular and general; Collective and non-collective; Absolute and relative; Concrete and abstract; and, Positive, negative and privative.

Simple and Composite Terms:

One worded terms are called simple terms. Examples: Agra, cat, library, etc. Many worded terms are called composite or complex terms. Examples: highest mountain peak, railway station, group of commandos, spring flowers, Royal Bengal tiger, good humor, wise men of Nottingham etc. Singular, General and Collective Terms: When a term designates one individual or an object it is called singular term. Examples: Agra, Qutub Minar, etc. When a term designates many objects or individuals it is called general term. Examples: trees, rivers, men, etc. A term applicable only to a collection of objects, but not to any individual member, is called a collective term. Examples: library, Indian army etc. The term ‘library’ is applicable to a set of books, but you cannot pull out a book and call it a ‘library’. Similarly the term ‘Indian army’ refers to a set of soldiers and officers, but we cannot refer to one soldier or officer from the set as ‘Indian army’.

Absolute and Relative Terms:

A term is an absolute term when its meaning is understood with the help of that term only. Examples: cows, river, etc. A term is relative when its meaning is understood with the help of some other terms. Examples: grandfather, wife, etc.

Concrete and Abstract Terms:

A concrete term refers to objects or a class which exist in space and time and which can be perceived. Examples: car, Eden garden, stars, fish etc. An abstract term refers to qualities or entities which cannot be perceived. Examples: God, demon, love, honesty, virtue, happiness, centaur etc.

Positive, Negative and Privative Terms:

A term is positive when it refers to the presence of qualities. Examples: good, happy, big, train, flowers etc. A term is negative when it refers to the absence of qualities. Examples: non-violence, non-cooperation, non-vegetarian etc. Note that a negative term does not imply an opposite term in the sense of 'black-white', 'hot-cold', 'rich-poor' etc. Prefixes like un-, dis-, as in 'undesirable', 'unbelievable' etc., also do not make a term negative; neither do suffixes like -less, 'powerless', 'homeless' etc., make term negative. It is the meaning that determines its character. A term is privative when it refers to the deprivation of a quality related to comfort or pleasure. Examples: The term 'deaf' deprives an individual of the 'pleasure of hearing'. 'Pain' deprives one of being painless.

Check Your Progress 2

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1) What do you mean by denotation and connotation of terms?

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2) Write in detail about the classification of terms.

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2.7 LET US SUM UP

In this unit we have attempted to understand various linguistic usages with which a student of logic must be familiarized. We started the discussion by distinguishing concepts, words and terms. It was said that a concept is a general idea, while a word consists of a letter or combination of letters conveying some meaning. A term, on the other hand is a word or a combination of words which by itself is capable of being used as subject or predicate of a logical proposition. Logicians use the words extension and intension. Some logicians try to give a mathematical form of expression to the quantitative relation between connotation and denotation. They say, 'connotation and denotation vary in inverse ratio'. Further ,terms can be classified as simple and compose terms, singular and general terms, collective and non-collective terms, absolute and relative terms, concrete and abstract terms, positive, negative and privative, and finally connotative and denotative terms.

2.8 KEY WORDS

Terminology: Terminology is the study, among other things, of how the terms come to be and their interrelationships within a culture.

Criterion: Criterion, in Logic, is an issue or standard used regarding the starting point of an argument or knowledge.

2.9 QUESTIONS FOR REVIEW

1) Distinguish between word, concept and term.
2) Explain different classification of words.
3) What do you mean by denotation and connotation of terms?
4) Write in detail about the classification of terms.

2.10 SUGGESTED READINGS AND REFERENCES

- Nath Roy, Bhola. Text Book of Deductive Logic. Calcutta: S.C.Sankar & Soul Private Ltd, 1984.
- Felice, Anne. Deduction. Cochin: 1982.
- Cook, Roy T. "Intensional Definition". In A Dictionary of Philosophical Logic. Edinburgh: Edinburgh University Press, 2009. 155.

2.11 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress 1

1. A concept is a general idea .A word consists of a letter or combination of letters conveying a determinate meaning. A term is a word or combination of words which by itself is capable of being used as subject or predicate of a proposition.

2. Words are classified as categorematic, syncategorematic and a-categorematic words. Categorematic words are those which can by themselves be used as terms without the help of other words. Syncategorematic words are those which cannot be used independently as terms, but which can only be used along with other words, e.g. of,

with, and, the, come etc. Acatgegorematic words are words which express only exclamatory feelings or emotions.

Check Your Progress 2

1. Denotation means number, or the reference of a term. It is also known as extension because it shows the extent or range of objects to which a term is applied. The function of suggesting qualities possessed by this objects is known as connotation.

2. Terms can be classified as: Simple and composite terms Singular and general terms Collective land non-collective terms 10 Absolute and relative terms Concrete and abstract terms Positive, negative and private terms Connotative and non-connotative terms

UNIT 3: DEFINITION AND DIVISION

STRUCTURE

3.0 OBJECTIVES

Logic deals with thought, and thoughts are always expressed in language in which different words we use are expected to convey proper idea. If there are no fixed ideas, it would be difficult to understand what one means by a word. In such a situation error and confusion will be the result. For example, a lawyer and a doctor do not define the term ‘man’ in the same sense. Their definitions are bound to vary. We define a term according to the interest we have in it. But logic deals with correct thinking. Our thoughts can never be correct unless we determine the meaning of each term through correct language. Each term must be understood in its proper sense. The tools which logic uses to achieve this purpose are definition and division. Therefore, the unit aims at introducing the students to:

• To know the correct thinking

• To discuss the correct language

• To know the correct knowledge of definition and division

In the previous unit we have seen that a term may be defined in two ways:

1) by reference to its denotation and
2) by reference to its connotation. Explanation of a term is with reference to its denotation and it is known as division, and explanation of a term with reference to its connotation is known as definition. In this unit we undertake a detailed study of definition and division.

3.1 INTRODUCTION

Language is a very complicated instrument, the principal tool for human communication. But when words are used carelessly or mistakenly, what was intended to enhance mutual understanding, may, in fact hinder it. Our instrument thus becomes our burden. This can happen when the words used in a discussion are ambiguous or emotionally loaded. True, most controversies involve much more than words, but sometimes conflict turns chiefly on and unsuspected difference in the ways the parties are using some key terms whose different senses may be equally legitimate, but must not be confused. Then it is useful to be able to specify or explain the different senses of the ambiguous term. Definitions can effectively resolve disputes that are merely verbal by exposing and eliminating ambiguities. Definitions are essential to expose and prevent fallacies of ambiguity and reasoning. We shall begin first by examining the nature of definition.

1. Introduction: kinds of division

Division used to be a central topic in logic. The logic in question was an Aristotelian style of logic, which was taught and studied prior to the modern logic of Frege, Russell, and others i.e. prior to about 1920. For example, J. J. Toohey (1918) An elementary handbook of logic has a chapter on Division. See also Jevons (1883, Section II), and Parry and Hacker (1991, Chapter 6). Toohey (1918, Chapter XVI) distinguishes:

Logical division, which is the resolution of a class into the subclasses that compose it (e.g. triangles into obtuse-angled, right-angled, and acute-angled)

Physical division, which is the resolution of an individual thing into the physical parts which compose it (e.g. a particular sword into its hilt and its blade)

Metaphysical division (or mental distinction), which is the resolution of 'objects' into the attributes which they possess (e.g. man into rational, sentient, organic, corporeal, warm-blooded, mortal etc.)

Verbal division, which is the resolution of a word which is a homograph into the synonyms which compose it (e.g. 'palm' into 'palm (kind of tree)' and into 'palm (part of hand)')

Many other similar kinds of division might be imagined (e.g. mathematical division, which might include a partitive factoring of a natural number into its component primes).

Division is obviously important to Knowledge Organization. Typically, an organizational infrastructure might acknowledge three types of connecting relationships: class hierarchies, where some classes are subclasses of others, partitive hierarchies, where some items are parts of others, and instantiation, where some items are members of some classes (see Z39.19 ANSI/NISO 2005 as an example). The first two of these involve division (the third, instantiation, does not involve division — see below). Logical division would usually be a part of hierarchical classification systems, which, in turn, are central to shelving in libraries, to subject classification schemes, to controlled vocabularies, and to thesauri. Partitive hierarchies, and partitive division, are often essential to controlled vocabularies, thesauri, and subject tagging systems. Partitive hierarchies also relate to the bearers of information; for example, a journal would typically have its component articles as parts and, in turn, they might have sections as their parts, and, of course, components might be arrived at by partitive division (see Tillett 2009 as an illustration). Finally, verbal division, disambiguating homographs, is basic to controlled vocabularies.

Thus Division is a broad and relevant topic. This article, though, is going to focus on Logical Division.

2. The basics of logical division

Logical division concerns collections, and sub-collections of those collections. It concerns the family of concepts exemplified by sets, classes, kinds, types, sorts, and similar concepts, and it concerns the subclass-superclass relationships (or subtype-type relationships, or subset-set relationships etc.). There have been many different specific theories of sets, classes, kinds, types, and the like. What is needed to discuss logical division in general is a certain accommodation and gentleness with respect to these different concepts. In this article, the word class will be used to cover any of class, kind, type, sort, set etc. Then, subclass-superclass will be the primary relation of interest.

Logical division divides a class into some of its subclasses, then some of those subclasses into some of their subclasses, and so on, a finite number of times. In general, any class will have many subclasses, but logical division is typically interested only in collections or families of subclasses that "divide up" the original class i.e. the subclasses resulting from a step of division need to be disjoint and not have members in common. A single step of logical division produces something akin to a partition of the original class, then the next steps produce partitions of those partitions, and this process continues in a like manner. An important distinction within the theory of classes is that between intension and extension, what Frege calls Sinn (sense) and Bedeutung (reference) (Tichý 1988). To conceive of, or to define, a class intensionally is to give a property, or concept, which characterizes it. To conceive of, or to define, a class extensionally is to give a listing of its members. Suppose, to give an example, that every red object in the world was also round, and every round object was also red; then the classes red and round would be co-extensive, they would have the same members; in which case, conceiving of classes extensionally, just as listings of their members, the classes red and round would be the same class; there would just be the one class. In contrast, conceiving of the classes intensionally, the classes are different and there are two of them, the property red is a color and the property round is a shape and even if, in our world, everything that was red was round and vice versa, there would be, or could be, other worlds, other possibilities that we can conceive of, in which there are some red objects which are not round, or round objects that are not red.

Logical division has sometimes been treated extensionally, in terms of dividing up listings of members, and sometimes treated intensionally, in terms of dividing up classes produced by properties or characteristics (see Marradi 1990, Howton 2010 for further discussion). A consideration here is whether division is going to be used on classes in mathematics, logic, and other a priori and necessary areas, or on classes in science and everyday matters of fact. Mathematics is extensional; for example, what prime numbers there are, there simply are — there is not some kind of alternative reality in which there are a few more or a few less prime numbers. In contrast, science and everyday matters of fact, are intensional; for example, the class of 19th century mathematical logicians has, as a matter of fact, Frege as one of its members, but it might have been that Frege chose a different line of business, in which case, the same (intensional) class 19th century mathematical logicians would have had a different extension. It is almost always better to treat division using an intensional conception of classes, but in mathematics, logic, and some other areas, an extensional conception can be adequate. With some historical writers, for example Plato, it is not entirely clear whether the division is intensional or extensional (Howton 2010) — to be fair, Plato was writing 2000 years before Frege.

If intensional division is used, the technique would usually be that of adding properties or conditions to the higher level properties; for example, the class animals can be divided into the subclasses animals and warm-blooded and animals and not-warm-blooded, and this is just adding or conjoining the properties of being "warm-blooded" or "not-warm-blooded" to the base property of being an animal.

The division of a class into subclasses produces only classes, and division of those subclasses produces only further classes. Within the domain of logical division, there is no interest in, or theory of, instances i.e. members of those classes. So, for example, logical division may address whether the class man and the class horse are subclasses of the class animal, but there is no interest whatsoever in, for example, whether the individual Socrates is an instance of the class man or of the class horse or of any other class. There is a caveat that can be given here. Nowadays we are perfectly sound on the distinction between subclass and instance (or member); for example, the class old man is a subclass of man, and Socrates is an instance of old man (and of man), but Socrates is not a subclass of old man (nor of man). But this distinction only really comes clear in the 19th century with Cantor and set theory. So pre-19th century materials on class and instance may or may not be perspicuous on the distinction.

In a finite class hierarchy produced by division, the structure is that of tree, i.e. a rooted connected acyclic graph (see Diestel 2012 for an explanation of these terms), and so there are nodes or classes or species that do not have children. These are the "leaves" of the tree — they are the infima species. Division stops at the leaves. Somewhat similarly in the other direction, there is a node or class or species that does not have a parent class. This is the "root" of the tree — it is the summum genus. Division starts with the root class. There is the notion of level of a class or node in a tree, and this is identified by the number of links between the root of the tree and the class. Sometimes, for example in eighteenth century biology, the levels can have particular names of their own, e.g. "kingdom", "phylum", "family" (Linnaeus 1758).

It is possible to view classification and division as companions or counterparts. If so, division would be "top-down". Indeed, division has been referred to as "downward classification" (Mayr and Bock 2002, Mayr 1982). The starting point would be a very general class which would successively be narrowed until a suitable classification class was reached. The whole division and narrowing process produces a classification system, a tree of classes. In contrast to division, the plain act of classification, i.e. the action or process of putting items in classes or categories, would normally be "bottom-up". The starting point would be one or more items or individuals, which needed classifying, and they would be classified by putting those with commonalities as members of a suitable narrow class and proceeding upward. Care is needed with the viewpoint that division and classification are much the same, apart from being in different directions The process of classification requires identifying suitable classes with the items as members i.e. it requires consideration of membership or instantiation. In contrast, the process of division has no connection whatsoever with membership and instantiation.

History

There are four important philosophical figures, historically, that set the scene for logical division: Plato (circa 450 B.C.E.), Aristotle (circa 400 B.C.E.), Porphyry (circa 270 C.E.), and Boethius (circa 500 C.E.). And there is one prominent scientist that should be mentioned: Linnaeus (circa 1740 C.E.).

Plato in the *Sophist* seeks a definition of the form or class or kind sophist, and there is a specific dialectical method that he advocates (Gill 2016, 2010, Howton 2010). It is that of starting with a very general kind, then using division to divide that kind in two, then repeating this process over and over until the exact kind sophist was met. It was as though one were travelling on a journey down a road, and every time the road had a fork, one path was chosen, until the destination was reached. The meeting of a fork, and the choosing of one of the (usually) two possibilities is the technique of division. At the destination point, the process was reversed, or the route retraced, and all the division properties were accumulated together as the definition of sophist. Plato used the same technique in the *Statesman* to define the kind statesman, and the approach was assumed to be general (Gill 2016, 2010, Howton 2010).

Notes

The notion of definition in use here is not that of explaining the meaning of a word (say the word *sophist*) rather it is that of capturing what it is to be an X (in this case, to be a sophist). It is to grasp what is essential.

Aristotle also offered a theory of definition, in *Topics* (Smith 1997). This rests on Aristotle's theory of classification (Berg 1982, Smith 2016). In this, it is a *species* that is defined, and a species is defined by means of a *genus* and a *difference*. So, for example, a classification fragment from the history of biology might be

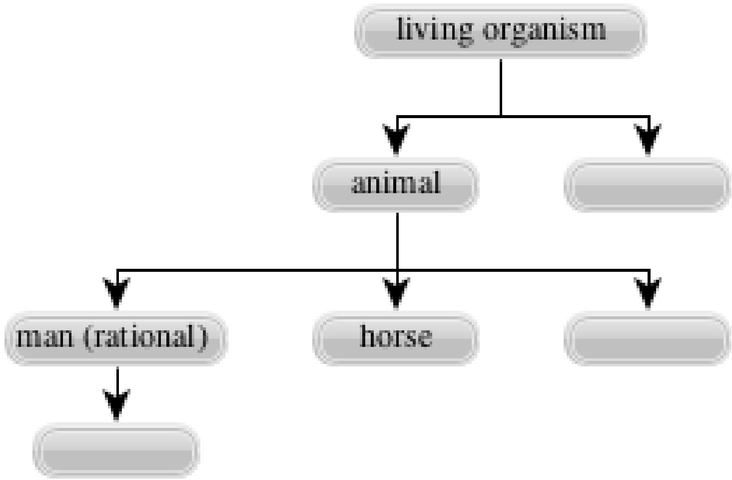

Figure 1: A classification fragment

This diagram is illustrating a partial classification hierarchy among classes; so, for example, animal is a subclass of living organism and a superclass of man and horse. An alternative way of describing this is to say that animal is a child class of living organism and a parent class of both man and horse.

All of the members of a classification hierarchy which are children are *species* — so horse is a species. Any member of the hierarchy which is a parent class is a *genus* — so, animal is a genus. Then the child species of a specific genus are separated one from another, i.e. from their sibling species, by means of the *differences*. For example, Aristotle thought that what was characteristic of man was that man had the capability of reasoning i.e. was rational. (The other species' differences are not illustrated in the diagram.)

It is the species that are *defined*, and they are defined by identifying their parent i.e. their genus and putting that together with the *differentiae* which is the difference which separates or distinguishes them from their siblings. So, an example definition is

man = $_{\text{df.}}$ animal having the capacity to reason

All such definitions have the form

<species> = $_{\text{df.}}$ <genus> <difference>

When a class or species is defined this way, the defining properties on the right hand side of the definition are *essential* properties that all instances of the species must have. Definition, Aristotelian definition, is the definition of a species by means of its genus and difference (Berg 1982, Smith 2004). (Just as a historical note, Aristotle did not use classification diagrams, that came later inspired by Porphyry, and also Aristotle did not use the word *species*, that also came later, with Porphyry and Boethius.)

Like Plato, Aristotle sometimes used the method of division to produce essential definitions. However, he was critical of Plato's approach to division. His view was that Plato-style division could be used as a heuristic to discover essential definitions, but the method was not strong enough as a method of proof to prove that the tentative essential definitions were indeed truly correct and that they captured the relevant essences. Aristotle argues this in *Posterior Analytics* II 3-10, and *Prior Analytics* I 31 (Smith 1989, 2016) and Howton (2010) provides a discussion.

Porphyry was a commentator on Aristotle, in particular Porphyry's *Isagoge et in Aristotelis Categorias commentarium* is an introduction to logic and a commentary on Aristotle's Categories. *Isagoge* became the standard text for logic in the Middle Ages; indeed, it served as a basic introductory text in philosophy for 1000 years (Eyjólfur 2015). In it, Porphyry introduced the "Tree of Porphyry" and these are classification trees, produced by division, where the division is a bifurcation (or dichotomy or exhaustive division) at each step (Verboon 2014, Hacking 2007). Jevons (1883, 232) provides the example

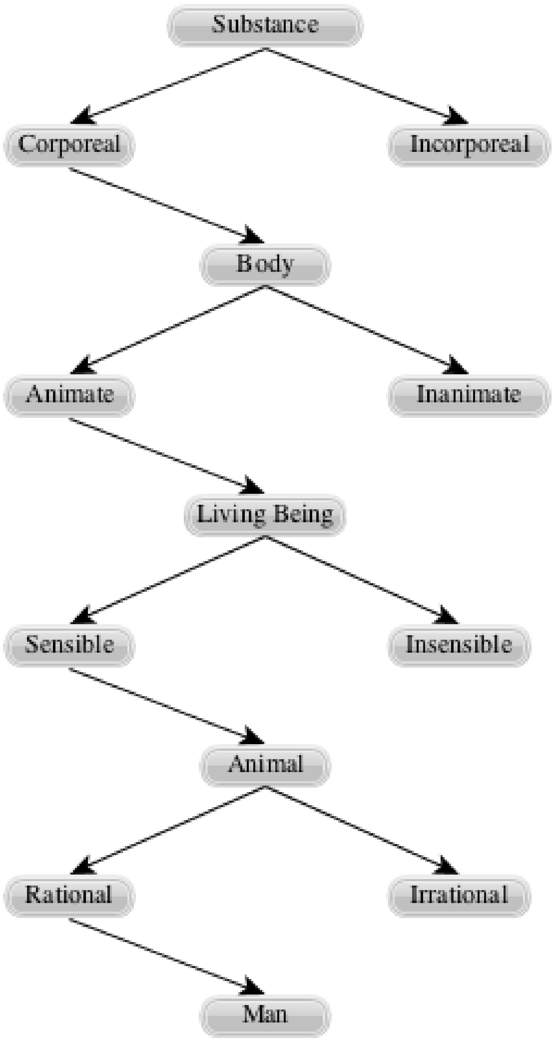

Figure 2: Jevons's example of a Tree of Porphyry

Jevons has presented the tree in a certain way, tidying up much older diagrams. He has omitted Latin annotations. He has omitted potential children of the negation classes e.g. Incorporeal does not have any children that are shown in the diagram. Then what seem to be the immediate children of a genus are in fact sometimes the differentiae which are then collected back into the real genus immediate child, for example, animal descends left to rational, which is the difference, and animal and rational are collected together to form man which is the child of animal.

Boethius was also an influential commentator on Aristotle's work (Arlig 2015). He was the main medieval authority on division, and he wrote a handbook on division (Magee 1998). Boethius also provided a commentary on Porphyry's *Isagoge* and translated it. It is in the manuscripts of those translations that the first diagrams of the Tree of Porphyry appear (Verboon 2014). So Boethius is important in conveying these ideas to a wider audience.

In the history of science, Linnaeus was probably the most prominent scientific classifier of the natural world. He used "binomial nomenclature", which is the identifying of classes by means of genus and species (Linnaeus 1758). At least some of the time, Linnaean classification structures were produced by logical division, especially logical division using dichotomy or bifurcation e.g. into warm-blooded and not-warm-blooded, into feathered and not-feathered, etc.

3.2 NATURE OF DEFINITION

Classical logicians have tried to define terms. The term to be defined is called definiendum and its definition is called definien. According to them, definition aims at unfolding the meaning of a term. It is the explicit statement of complete connotation of a term. The connotation of a term consists of essential attributes of the term. The purpose of defining a term is to understand the meaning of a term. For example, while defining man, rationality and animality are the two essential qualities which are considered. Hence man is defined as a rational animal. Popularly definition is divided into two types; verbal and real definition. The time honoured rule of definition is that it is per genus at differential, i.e., a statement of the connotation of the proximate genus and the differentia of the term. In other words while defining a term one has to state the genus and the differentia. Genus means the class and the differentia means the distinct quality unique to definiendum and therefore differentia means definien. A definition consists in stating first the class to which the definiendum belongs and then state the definien. It must be noted that this order is irreversible. In other words, in defining a term, we first of all decide to what class of things it belongs and then, we mark the attribute or group of attributes, which distinguish it from other members of that class. For example, while defining man as a rational animal, it is meant that man possesses the attributes of 'animality', belonging to its proximate genus animal and the differentia, rationality. It is the differentia because it belongs to none other than man. Similarly, when we define plane triangle as a figure bounded by three straight lines, the proximate genus is figure and the differentium is the attribute of being bounded by three straight lines. (This view of definition is based on a presupposition that there is a highest class followed by lower classes. The highest class is known as the summum genus and the lowest class is known as the infima species. Aristotle and the medieval logicians firmly believed that the smaller class is included in the bigger class. This theory of logic of Aristotle is complementary to his theory of biology. Aristotle believed that there are natural classes; genus, species and the entire animal kingdom including the vegetative kingdom can be classified on the basis of genus- species relation. But this type of ordering of terms is not to be found in the domain of language.) Attributes which we consider in a definition fall into three groups, viz. those which constitute the connotation of a term, those which follow from the connotation (known as properties) and those which neither constitute the connotation nor follow from the connotation (called accidents). If one states the entire connotation, i.e., proximate genus and differentia, we have the definition of the term. If, on the other hand, we enumerate its properties or accidents or merely a part of the connotation we have a description. A description is different from definition. While definition states the entire connotation, description states properties, accident and sometimes a part of the connotation. Definition is scientific while description is popular. The object of the former is to make our ideas of things distinct and clear while the object description is to furnish a rough and ready means of identifying an object. There are different kinds of definitions

1) Ostensive definition: When we explain the meaning of a term by pointing or showing the corresponding object, it is called ostensive definition. For example, when a little child asks what a ball is, the best way to teach him the correct use of this term is to

show him a physical object known as ball. Language is not needed to explain the meaning of a term. Thus ostensive definition is non-verbal in nature. All physical objects can be explained in this manner.

2) Denotative definition: When a term is defined by referring to the denotation of that term, it is called denotative definition. For example, to know the meaning of the term Scripture one can cite the names like the Vedas, the Bible, the Guru Grant Sahib, etc. Such definition is called verbal and denotative. Sometimes one can make use of the extension of the term to define it. This way of defining term is called extensive definition.

3) Connotative definition: When we explicate the connotation of a term, it is called connotation or connotative definition. It explicitly states the connotation of a term. Thus definition should be per genus et differentia', which has been stated earlier.

3.3 RULES OF DEFINITION AND FALLACIES

A connotative definition should conform to the following rules;

Rule I:

The definition should state the entire connotation of the term, neither less nor more. The connotation of a term consists of common and essential attributes. Therefore, while defining a term we should avoid inessential attributes. Even common attributes may be avoided unless they are at the same time essential attributes as well. Example: "Man is a rational animal" i.e., Man is that which has animality and rationality. Similarly 'a plane triangle is figure bounded by three straight lines'. If this rule is violated, fallacies by stating either more than the connotation or less than the connotation are committed. This suggests that the fallacy created by not following Rule I is of two types. Let us examine each separately.

A. If the definition states more than the connotation, the additional attribute would be either

1) superfluous or

2) an inseparable accident or

3) a separable accident, leading to the fallacies of Redundant, Accidental and Too Narrow definitions.

1. Fallacy of Redundant definition: If the additional attribute be a property we have the fallacy of redundant definition. The additional attribute is a common attribute but not an essential attribute. Hence it is superfluous to state it in a definition. For example, the definitions of triangle as "A plane figure, bounded by three straight lines, and having three angles" is a redundant definition because, the attribute of "having three angles" is superfluous.

2. Fallacy of Accidental definition: If the additional attribute be an inseparable accident, we have the fallacy of accidental definition. For example, the definition of man as "A laughing rational animal " is an accidental definition, because the attribute laughing even though as an attribute is found at times in men, is not a part of the connotation of the term man. 3. Fallacy of Too narrow definitions: If the additional attribute be a separable accident we have the fallacy of too narrow definition, because it is no longer applicable to its whole term but only to a part of it. For example the definition "Man is a civilized rational animal" is too narrow as the attribute civilized does not belong to all men. Similarly, if we were to define a triangle "as a plane figure enclosed by three equal straight lines", it is not sufficiently extensive.

B. Now let us attend to the next section. If the definition states something less than the connotation we have the fallacy of too wide definition. It is so called because it will apply to a greater number of things than are included in the denotation of the term defined. For example, "diamond is carbon" is too wide because it not only applies to diamond but also to all things made up of carbon.

Rule II: The definition should be clearer than the term defined and should not, therefore, be expressed in figurative, ambiguous or obscure language. Violation of this leads to the fallacies of figurative and obscure definitions. Examples for figurative definitions:

1) "Childhood is the morning of life"

2) "Necessity is the mother of invention" Example for obscure definitions: A girl is a perpendicular biological phenomenon in short skirt.

Rule III: The definition should not contain the term defined, or a synonym of it. Violation of this rule leads to the fallacy of circular definition. For example "the sun is the center of the solar system". Here the term solar system already presupposes Sun that is to be defined.

Rule IV: A definition should not be negative when it can be affirmative. A definition should positively state what the term means if it is possible to make an affirmation about it. A negative proposition merely states what a term does not mean. Violation of this rule leads to the fallacy of negative definition. Examples:

1) "Mind is not matter."

2) "Failure is but want of success." When we find it difficult or absolutely impossible to define a term, the so-called negative definition may come to one's aid to describe the entity. In Indian philosophical tradition while defining Brahman, the Advaita resorts to this type of definition presenting well the incapability to connote Brahman positively. Indian logicians however took objections to this type of definition. To conclude, a definition should be a precise, clear and adequate, and should not be tautologous, redundant or negative.

The standard treatment of informal fallacies used in many introductory university logic courses has undergone substantial revision over the last 50 years. For convenience, we will three consequent areas of non-deductive inquiry which involve non-formal reasoning rather than formally valid arguments; these include (1) informal logic (including academic critical thinking, conductive arguments and inductive arguments), (2) dialectical logic (including pragma-dialectical discourse analysis and pragmatic argumentation theory), and (3) rhetoric (including persuasion).

Critical thinking differs from informal logic in that critical thinking emphasizes various intellectual activities for problem solving in accordance with rational criteria, whereas informal logic is narrower in focus and emphasizes traditional argument interpretation and evaluation of argument everyday language, dialogue, and discourse. Pragma-dialectical discourse sets rules for rational critical discussion, and the speech acts which violate these rules are viewed as fallacies. Pragmatic logical argumentation evaluates defeasible normative reasoning within a dialectical context by means of argumentation schemes, mappings, and appropriate contextual standards of proof. In our course the informal logic approach is emphasized.

In 1970 C.L. Hamblin pointed out that the standard treatment of fallacies remained much the same as the thirteen fallacies pointed out by Aristotle in his Sophistical Refutations. Hamblin decries the standard treatment:

"[I]n most cases, I think it should be admitted, is as debased, worn-out and as dogmatic a treatment as could be imagined — incredibly tradition-bound, yet lacking in logic and historical sense alike, and almost without connection to anything else in modern logic at all."

And Douglas Walton cautions:

"Indeed, it has been shown that many of the so-called 'fallacies' are, in some instances, not incorrect arguments at all, but reasonable kinds of argument or reasonable kinds of criticisms of arguments."

Currently, defeasible reasoning and informal fallacies are analyzed in accordance with argumentation schemes and pragmatic theories of dialogue. Moreover, on many occasions, "traditional informal fallacies" can be set out and explained as acceptable arguments.

E.g., many traditionally defined informal fallacies such as ad hominem, ad verecundiam, tu quoque, ad ignorantiam, slippery slope, composition, division, and others can be contextually effective and legitimate arguments.

Wreen here

Decades ago, F.H. van Eemeren and R. Grootendorst pointed out:
"Referring to mitigating circumstances which make a fallacy no fallacy after all, however, does not solve anything. … [T]he detection and identification of fallacies becomes very much ad hoc: Each case has to be examined on its own merits … an adequate theory of fallacies is then out of the question.

Mistakes in arguments present in critical disputes are not limited to the errors of invalidity and unsoundness, but, as we will see, errors arise from the meaning or "content" of the statements used. The legitimacy of informal logic as a proper discipline is grounded on the presupposition that not all instances of ordinary language reasoning can be accurately translated into the abstract a priori structures of formal logic. In other words, the possibility of informal logic as a logical theory is based on the rejection of Sir William Hamilton's logical postulate as applied to formal logic:

"The only postulate of Logic which requires an articulate enouncement is the demand, that before dealing with a judgment or reasoning expressed in language, the import of its terms should be fully understood; in other words, Logic postulates to be allowed to state explicitly in language all that is implicitly contain in the thought."

Some logicians, e.g., Peter Ramus, Francis Bacon, and John Locke, among others, did not assay the treatment of informal fallacies because they emphasized that logic is concerned with correct reasoning.

Yet, unless we are aware of some of the mistakes that are likely to be made, i.e., unless we know how to avoid typical mistakes in thinking, we are unlikely reason well.

No one is particularly satisfied with the traditional treatments of fallacies — many fallacies cannot be categorically well-defined. Thus, it may well be impossible to give a systematic treatment of fallacies, in part because different methods of logic have been adopted and in part because the nature of errors in reasoning are not always due to grammar and forms of language.

Augustus De Morgan writes in his Formal Logic: "There is no such thing as classification of the ways men arrive at an error: it is much to be doubted whether there ever can be."

H.W.B. Joseph says in his Introduction to Logic, "Truth may have its norm, but error is infinite in its aberrations, and they cannot be digested in any classification. The same inconclusive argument may often be referred at will to this or that head of fallacies."

As an example of a fallacy which may be identified in either of two ways, consider the fallacy of the ambiguous middle term in syllogistic logic, also termed the four term fallacy and normally described as a formal fallacy. The same argument can also be identified as a fallacy of equivocation which is an informal fallacy. Augustus De Morgan provides this example of a four term fallacy with an equivocation of the term “criminal action”:

“All criminal actions ought to be punished by law.
Prosecutions for theft are criminal actions.
Therefore prosecutions for theft ought to be punished by law.”

The first occurrence of the phrase “criminal action” refers to “the commission of a crime” and the second refers to a “legal proceeding.” Either fallacy can be cited as the reason the argument is fallacious.

No unified theory of fallacy has been proposed, except in an exceedingly superficial manner by negative definition. I.e., a fallacy is said to be incorrect reasoning. This negative definition of course depends upon having a complete and consistent explication of the meaning of proper reasoning.

However, most logicians do not consider just any error in reasoning a fallacy per se; the term “fallacy” is reserved by many logicians for recognizable errors of a specific kind. Even so, some accounts of informal fallacies are cluttered with countless minor variations of traditional fallacies. Specific informal fallacies often are not definable in terms of some specific or common trait but are instead characterized by various family resemblances. For this reason, the completion of the development of a standard taxonomy of informal fallacies is implausible.

So, most likely, what we call ‘fallacy’ has no common single meaning, and the characterization of the uses of this word fits Ludwig Wittgenstein's notion of “family resemblance” — where “we see a complicated network of similarities overlapping and criss-crossing; sometimes overall similarities, sometimes similarities of detail.”

Francis Bacon wrote in a similar regard:

But these small wares and petty points of cunning are infinite; and it were a good deed to make a list of them, for that nothing doth more hurt in a state than that cunning mean pass for wise.”

But most fallacies and related infelicities of persuasion occurring in everyday discourse much in their complexity that attempting to name as many of possible would prove unprofitable.

Even though there exists no agreed upon theory of fallacy, informal fallacies in this course will be more or less organized traditionally. An attempt is made to incorporate much like the standard treatment of fallacies with a view toward informed practicality in everyday reasoning.

Let us start with the question, “what is a fallacy?” Following Richard Whately, the first English logician to to base the subject of fallacies on logical principles by dividing fallacies into logical or formal and material, many logicians also define “fallacy” as some form of deceptive reasoning. Richard Whately writes:
“By a ‘Fallacy’ is commonly meant ‘any deceptive argument or apparent-argument, whereby a man is himself convinced, — or endeavors to convince others — of something which is not really proved.”

And Jeremy Bentham writes:

“By the name fallacy it is common to designate any argument employed, or topic suggested, for the purpose, or with a probability, of producing the effect of deception …

Indeed, the conventional definition of “fallacy” expressed in the Oxford English Dictionary is “A deceptive or misleading argument, a sophism.”

The traditional view of fallacy, closely related to this view, is the oft-used contemporary definition: “an argument which seems to be valid, but really is not.” However, several serious problems with this definition for logic are evident.

The definition is psychological; it turns on whether an individual happens to be misled by an argument. William Spalding points out deception are not essential to the definition of fallacy:

“The name is sometimes … used to deceive. But the intention is a point of secondary importance in the theory of fallacies …

What could count as something being unqualifiedly “deceptive”? E.g. consider this oft-used fallacy of equivocation in several 18th century books:

Nothing is better than Heaven;

But a Penny is better than nothing;

∴ A Penny is better than Heaven.

Since few persons would regard the argument as valid, on the above definition of “fallacy,” the syllogism would not be treated as fallacious because it is not deceptive. Nevertheless, the fallacy committed here is formally the syllogistic four term fallacy and informally the fallacy of equivocation. The middle term “nothing” is being used in two different senses of the word: a positive sense in the first premise and a negative sense in the second premise.

The notion of validity is normally applied to deductive arguments only. So the above proposed definition of “fallacy” would not address the incorrectness or acceptableness of inductive, probabilistic, or informal arguments.

Moreover, in the case of petitio principii (circular reasoning), its fallacious aspect is not its deductive invalidity, but instead its deceptiveness. This is one reason many argumentation theorists argue that some informal fallacies are not necessarily viewed as arguments per se but instead viewed as deceptive techniques or rule violations. Additionally, the fallacy of complex question (the fallacy of many questions) is not prima facie an argument — it's couching an unwarranted presupposition within a question.

3.4 LIMITS OF DEFINITION

Following are the limits of definition: Summum genus cannot be defined. We have already seen that a definition should be per genus et differentiam. The summum genus, being the highest genus, cannot be brought under a still higher genus and therefore, it cannot be defined. Singular abstract names, which are names of elementary attributes, cannot be defined because there is nothing simpler or more elementary than what they are. For example, terms like equality, energy, etc. cannot be defined. Proper names and individual objects are indefinable. Proper names cannot be defined since they do not possess any connotation. Individual objects possess an infinity of attributes and therefore it is impossible to complete enumeration of all the attributes of them. Hence they too cannot be defined.

Check Your Progress 1

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1) What is definition and what are its different kinds?

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2) Explicate the rules of connotative definition.

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3.5 ON DIVISION

Aristotle and the medieval logicians tried to integrate definition with division. For them to define means to divide and vice versa. According to these logicians definition also means the division of bigger classes into proximate classes. Only classes are divided, individuals cannot be divided. Also the smallest class cannot be divided at all. The smallest or lowest class is known as the infima species. It should be remembered in this connection that Aristotle and the medieval logicians conceived language as consisting of only classes and sub-classes. But as a matter of fact this is not so. There are various types of words and terms in language which do not fit into this scheme. Let us understand more about Division. Division is the splitting up of genus or higher class into its constituent species or subclasses according to a certain principle. It is different from definition to the effect that the former is the analysis of the denotation of a term while the latter is the statement of its connotation. In fact logical division is division of a class into sub-classes and not a division of an individual thing into its different parts. To this extent it is different from natural division. There are various types of division viz.,

1) natural division,

2) metaphysical division and

3) logical division.

Classification and division which characterize biology is an example of natural division because it is easily discerned in nature itself. Man has no role to play in it. Metaphysical division is, on the other hand, the same as conceptual analysis. Substance- attribute, cause-effect, space-time, particular- universal, etc., illustrate metaphysical division. Both natural division and metaphysical division should be distinguished from logical division. Unlike the former two types it cannot be applied to an individual thing but only to a class of things. Logical division is the analysis of the extension of class terms. Here one splits a genus into its constituent species. It is closely connected with the process of classification of connotative definition. It is said that in defining we divide and in dividing we define. In order to define the term man, we state its genus animal and its differentia rational. This necessarily implies that the class of animal can be divided into two sub-classes from the standpoint of having or not having rationality, i.e., man and not-man. This way of defining involves division. Again, when we divide triangle into equilateral, isosceles and scalene taking into consideration the equality of sides, the definitions of these terms are evident, since their genus is triangle and the differentia are having three equal sides, having two equal sides and having unequal sides respectively. Thus division involves definition. When the term animal is divided, the term man is defined and when the term man is defined, the term animal is divided. Thus the primary aim of division is to make the meaning of the term clear.

3.6 RULES OF LOGICAL DIVISION

Logical division should abide by the following rules that follow from the very nature of the division.

Rule I:

The term to be divided must be a general term: This rule is evident from the very definition of logical division. It is only a class, which can be divided into its sub-classes. Division aims at giving us a complete idea of the extension of the term and all the sub classes constitute the extension of the class.

Rule II: Logical division must be according to one definite principle: If more than one principle is adopted we shall commit the fallacy of cross division. Division of students into tall, intelligent, fair and backbenchers is a case of cross division. Here the sub classes get mixed up together. In this case we have adopted four principles of division, namely intelligence, height, complexion and sitting habit. Consequently the very purpose of division is defeated.

Rule III:

The name of the class divided must be applicable to each of the subdivisions coming under it: All subclasses of a higher class belong to that class. Hence in every logical division the subdivisions may take the name of the class. Thus when the term man is divided into the subclasses, tall, short and medium sized, all these subdivisions being subdivisions of the class man, we can tell them to be tall man, short man and medium sized man. But division of man into head, hands, legs etc. is not a case of logical division. In these cases it is not possible to apply the term to each of the above parts, the 'head' is not man, 'hands' are not man.

Rule IV:

The sub-classes taken together exhaust the extension of the term defined: Division aims at giving us a complete idea of the extension of the term. Denotational definition is bound to be incomplete and hence extensional definition is preferred. In giving extensional definition we point out all the subclasses and if any sub-class is left out the division is incomplete. Dividing triangle into acute angled and right angled is incomplete because obtuse angle triangles are left out.

Rule V:

The sub-classes to which the term is divided must be mutually exclusive.

This follows from the rules that division must be always on single and fundamental principle. If the classes are not mutually exclusive we are sure that more than one principle have been adopted and the second rule has been violated. Thus the division of man into rich, tall and honest illustrates the fallacy of overlapping division. The subclasses are overlapping, not exclusive.

Rule VI:

In a continued division each step should divide a class or sub-class into its proximate sub-classes. This means division must not take a leap. If a logical division involves more than one step, it should be continuous, proceeding step by step without omitting any intermediate species. Violation of this rule leads to the fallacy of too narrow division. For example, rectilinear plane figures should not be divided immediately into such remote species as equilateral triangles, squares, parallelograms etc. It may also be noted that the rules mentioned above are all inter-connected. Hence the violation of any one of them may, at the same time, involve violation of other rules as well.

3.7 DIVISION BY DICHOTOMY

In many cases, it is difficult to assure us whether all the rules have been duly satisfied or not. Further, without material knowledge of the things denoted by the term it is not possible to have a correct form of logical division. In order to avoid these difficulties, a form of division called Division by dichotomy is suggested. Dichotomy literally means dividing into two. Division by dichotomy is illustrated when we divide a class into two complementary subclasses. For example, if we divide people of the world into Asians and non-Asians, then we have division by dichotomy. For someone familiarized with the rules of division it is clear that to assume ourselves whether all the rules have been duly satisfied or not seems an uphill task. Further, without the material knowledge of things denoted by a term, it is not possible to have a correct form of logical division. Since there is more than one principle of division, subclasses must not overlap and when taken together the subclasses should be equal to the class divided. Now it is clear that we are incapable of being certain that a particular logical division conforms to all the rules if we lack knowledge of the things denoted by the class to be divided. This kind of material knowledge is wanting in formal logic. Hence formal logicians conceived this kind of division. This division is done by mere form of the division. In this type even without my knowledge of the subject matter, which is being divided, we may be certain that the rules of division have been observed. Such a type of division is suggested to avoid difficulties that may arise as cited above (in fact some logicians consider division as a part of material logic). There cannot be more than one principle of division operating simultaneously. Therefore two subclasses can be obtained according to the principles of excluded middle and non – contradiction and therefore they must be mutually exclusive and together must be equal to the denotation of the class divided. In this way, the rules of division are observed, yet knowledge of the subject matter is not necessary. 9 Division by dichotomy has its strength and weakness. Its strength is that it ensures the completeness of a division in a formally perfect fashion as it is based on the laws of contradiction and excluded middle. At the same time, it is open to the serious objection that this type of division is superficial whereas what is expected of logical analysis is much deeper and clear division.

Check Your Progress 2

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1) What is division? Explain various kinds of division?

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2) Sate and explain rules of logical division.

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3.8 LET US SUM UP

In this unit we have taken up a very important problem in logic, namely the nature, types, functions and fallacies of definition along with logical division, which should necessarily form part of any course in logic. The problem of definition is clubbed with division since the course to be studied along with definition carries almost the same subject matter and their explanations are mutually dependent. We have seen that definition is the explicit statement of all the essential attributes connoted by a term. The purpose of defining a term, it was clarified, is to understand the nature of a term. After examining the nature of definition we have looked into the various rules of definition, violation of which would end up with definitional fallacies. It was noted that certain entities or terms are beyond the scope of definition and therefore, remain indefinable. Definition and division are interconnected issues. Different types of division viz., physical, metaphysical and logical were also discussed. Of these it was logical division that demands the attention of logicians. In division there are six rules, violation of which leads to fallacies of division. Since in many cases it is difficult to assure ourselves whether all the rules have been duly satisfied or not, logicians propose a type of division applicable in formal logic, namely division by dichotomy. Division by dichotomy is that type of division, which divides a class into two contradictory sub-classes, for example, the class of people on earth into Asians and not-Asians.

3.9 KEY WORDS

Meaning: Meaning is associated with connotation. It is precisely what we ought to understand.

Language: Language is the systematic creation and usage of systems of symbols referring to linguistic concepts with semantic or logical or otherwise expressive meanings.

Predicables: Predicables are the possible relations of the predicate to the subject. In this regard logician Porphyry spoke of five predicables, viz., genus, species, differentia, property and accidents. Genus and species refer to the denotative function of the terms; the other three refer to the connotative functions.

3.10 QUESTIONS FOR REVIEW

1) What is definition and what are its different kinds?
2) Explicate the rules of connotative definition.
3) What is definition and what are its different kinds?
4) Explicate the rules of connotative definition.

3.11 SUGGESTED READINGS AND REFERENCES

- Copi, Irving M. and Cohen, Carl. Introduction to Logic. New Delhi: Prentice-hall of India
- Private Limited, 1997
- Felice, Anne. Deduction. Coclin , 1982
- Nath Roy, Bhola. Text Book of Deductive Logic. Culculta: S.C. Sarkar and sons Private Ltd, 1984.
- Gibbon, Guy (2013). *Critically Reading the Theory and Methods of Archaeology: An Introductory Guide*. Rowman & Littlefield. ISBN 9780759123427.
- Potter, Karl H. (1991). *Presuppositions of India's Philosophies*, p.87. Motilal Banarsidass. ISBN 9788120807792. "Under-extension", "over-extension", and "mutual exclusion".
- Chakraborti, Chhanda (2007). *Logic: Informal, Symbolic and Inductive*, p.54-5. PHI Learning. ISBN 9788120332485. "Too wide", "too narrow", "incomprehensible", and "conflicting".
- Hughes, Richard E. and Duhamel, Pierre Albert (1966/1967). *Principles of rhetoric/Rhetoric principles and usage*, p.77/141. 2nd edition. Prentice-Hall. "Using in the definition itself the word to be defined or a close synonym of it."
- Schipper, Edith Watson and Schuh, Edward (1960). *A First Course in Modern Logic*, p.24. Routledge. "Incongruous", "circular", "negative", and "obscure or figurative".

- *"Circular Definition". Stephen's Guide to the Logical Fallacies.* Accessed September 2, 2014.
- *Johnson, Samuel (1755), "Oats", A Dictionary of the English Language*
- *Bunnin, Nicholas; Yu, Jiyuan (2008), The Blackwell Dictionary of Western Philosophy, John Wiley & Sons, p. 165, ISBN 978-0-470-99721-5*

3.12 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress 1

1. The connotation of a term consists of common and essential attributes included in the term and definition means an entire connotation of the term. The purpose of definition is to understand the nature of the term. There are different kinds of definition: ostensive, denotative and connotative definitions. Ostensive: defining by pointing to the object; denotative: definition by referring; connotative: defining per genus et differentia.

2. Rule 1: A definition should state the entire connotation of the term, neither less nor more.
Rule 2: A definition should be clearer than definiendum and should not, therefore, be expressed in figurative, ambiguous or obscure language.
Rule 3: A definition should not contain definiendum or a synonym of it.
Rule 4: A definition should not be negative when it can be affirmative.

Check Your Progress 2

1. Division is the splitting up of genus or higher class into its constituent species or subclasses according to a certain principle. These are various kinds of division: natural, metaphysical and logical. Natural: division among living beings, Metaphysical: conceptual analysis undertaken by philosophers, Logical: the analysis of the extension of class term.

2. Rule 1: The term to be divided must be a general term. Rule 2: Division must be according to one definite principle. Rule 3: The name of the class divided must be applicable to each of the Sub-divisions coming under it. Rule 4: The subclasses taken together exhaust the extension of the term defined. Rule 5: The subclasses to which the term is divided must be mutually exclusive. Rule 6: In continued division each step should divide a class or subclasses into its proximate sub-classes.

UNIT 4: ELEMENTARY NOTIONS AND PRINCIPLES OF TRUTH-FUNCTIONAL LOGIC

STRUCTURE

4.0 OBJECTIVES

The present unit takes a closer look at the truth-functional fragment of propositional logic. We try to show:

- To know how the truth-functional concepts of negation, conjunction, disjunction, material conditionality, and material biconditionality may be expressed in English as well as in symbols;

- To discuss how these concepts may be explicated in terms of the possible worlds in which they have application; and

- To discuss how the modal attributes of propositions expressed by compound truth-functional sentences may be ascertained by considering worlds-diagrams, truth-tables, and other related methods. In effect, we try to make good our claim that modal concepts are indispensable for an understanding of logic as a whole, including those truth-functional parts within which they seemingly do not feature.

4.1 INTRODUCTION

Language is a very complicated instrument, the principal tool for human communication. But when words are used carelessly or mistakenly, what was intended to enhance mutual understanding, may, in fact hinder it. Our instrument thus becomes our burden. This can happen when the words used in a discussion are ambiguous or emotionally loaded. True, most controversies involve much more than words, but sometimes conflict turns chiefly on and unsuspected difference in the ways the parties are using some key terms whose different senses may be equally legitimate, but must not be confused. Then it is useful to be able to specify or explain the different senses of the ambiguous term. Definitions can effectively resolve disputes that are merely verbal by exposing and eliminating ambiguities. Definitions are essential to expose and prevent fallacies of ambiguity and reasoning. We shall begin first by examining the nature of definition.

4.2 TRUTH-FUNCTIONAL OPERATORS

The expressions "not", "and", "or", "if... then . . . ", and "if and only if may be said to be sentential operators just insofar as each may be used in ordinary language and logic alike to 'operate' on a sentence or sentences in such a way as to form compound sentences. The sentences on which such operators operate are called the arguments of those operators. When such an operator operates on a single argument (i.e., when it operates on a single sentence, whether simple or compound), to form a more complex one, we shall say that it is a monadic operator.

Thus the expressions "not" and "it is not the case that" are monadic operators insofar as we may take a simple sentence like (5.1)

"Jack will go up the hill" and form from it the compound sentence (5.2) "Jack will not go up the hill"

or (more transparently) (5.3)

"It is not the case that Jack wil l go up the hill." O r we may take a compound sentence like (5.4)

"Jack wil l go up the hill and Jil l wil l go up the hill" and form from it a still more complex sentence such as (5.5)

"It is not the case that Jack will go up the hill and Jil l will go up the hill." 1 When an expression takes as its arguments two sentences and operates on them to form a more complex sentence we shall say that it is a dyadic operator. Thus , the expression "and" is a dyadic operator insofar as we may take two simple sentences like (5.1) "Jack wil l go up the hill" and (5.6) "Jil l wil l go up the hill" and form from them a compound sentence such as (5.7) "Jack and Jil l wil l go up the hill" or (more transparently) (5.8) "Jack wil l go up the hill and Jil l will go up the hill." O r we may take two compound sentences like (5.2) "Jack wil l not go up the hill" and (5.9) "Jil l wil l not go up the hill" and form from them a still more complex sentence such as (5.10) "Jack wil l not go up the hill and Jil l wil l not go up the hill." T he expressions "or", "if .. . then . .. ", and "if and only i f are also dyadic operators. Dyadi c operators are sometimes called sentential connectives since they connect simpler sentences to form more complex ones. 2 1. Note that this sentence is ambiguous between "It is not the case that Jack will go up the hill and it is the case that Jill will go up the hill" and "It is not the case both that Jack will go up the hill and Jill will go up the hill." This ambiguity, along with many others, is easily removed in the conceptual notation of symbolic logic, as we shall shortly see. 2. Some authors like to regard "it is not the case that" as a sort of degenerate or limiting case of a connective — a case where it 'connects' just one sentence. We, however, will reserve the term "connective" for dyadic operators only.

Now each of the sentential operators cited above is commonly said to be truth-functional in the sense that each generates compound sentences out of simpler ones in such a way that the truth-values of the propositions expressed by the compound sentences are determined by, or are a function of, the truth-values of the propositions expressed by the simpler sentential components. Thus it is commonly said that "it is not the case that" is truth-functional since the compound sentence "It is not the case that Jack will go up the hill" expresses a proposition which is true in just those possible worlds in which the proposition expressed by its simple sentential component "Jack will go up the hill" is false, and expresses a proposition which is false in just those possible worlds in which the proposition expressed by the latter sentence is true; that "and" is truth-functional since the compound sentence "Jack will go up the hill and Jill will go up the hill" expresses a proposition which is true in just those possible worlds in which the propositions expressed by the sentential components "Jack will go up the hill" and "Jill will go up the hill", are both true, and expresses a proposition which is false in all other possible worlds; that "or" is truth-functional since the compound sentence "Jack will go up the hill or Jill will go up the hill" expresses a proposition which is true in all those possible worlds in which at least one of the propositions expressed by the sentential components is true, and expresses a proposition which is false in all other possible worlds; and so on. This common way of putting it gives us a fairly good grip on the notion of truth-functionality. But it is seriously misleading nonetheless. For it is just plain false to say of each of these sentential operators that it is truth-functional in the sense explained. We should say rather that each may be used truth-functionally while allowing that some at least may also be used non-truth-functionally. Let us explain case by case. The uses of "not" and "it is not the case that" It is easy enough to find cases in which the word "not" operates truth-functionally. When, for instance, we start with a simple sentence like (5.11) "God does exist" and insert the word "not" so as to form the compound sentence (5.12) "God does not exist" we are using "not" truth-functionally. The proposition expressed by the compound sentence (5.12) will be true in all those possible worlds in which the proposition expressed by the simple sentential component of that sentence is false, and will be false in all those possible worlds in which the latter is true. But suppose now that we start with a simple sentence, (5.13) "All the children are going up the hill" and insert the word "not" so as to form the compound sentence (5.14) "All the children are not going up the hill." This latter sentence is ambiguous. And the answer to the question whether the operator "not" is being used truth-functionally on (5.13) depends on which of two propositions (5.14) is being used to express. On the one hand, (5.14) could be used by someone to express what could better, that is, unambiguously, be expressed by the sentence

4.3 EVALUATING COMPOUND SENTENCES

Truth-functional compound sentences do not, of course, bear truth-values: no sentences do, whether they are simple or compound, truth-functional or not. Only the propositions expressed by sentences bear truth-values. Nonetheless there is a sense in which it is proper to speak of the "evaluation" of sentences. As we have seen, the truth-values of propositions expressed by truth-functional compound sentences are logically determined by the truth-values of the propositions which are expressed by the sentences which are the arguments of the truth-functional operators in those sentences. Evaluating a sentence consists in a procedure for ascertaining the truth-value of the proposition expressed by a truth-functional compound sentence given truth-value assignments for the propositions expressed by its sentential components. Each of the examples of truth-functional compound sentences considered in the previous section featured only one sentential operator and at most two sentential arguments — one argument in the case of the monadic operator "^ w , and two arguments in the cases of the dyadic operators " •", "v", "o", and " = ". It is time now to look at techniques for evaluating well-formed compound sentences which might feature any arbitrary number of truth-functional operators. Although in ordinary speech and in casual writing, we have little occasion to produce sentences with more than just a few operators in them, the special concerns of logic require that we be able to construct and evaluate compound sentences of any degree whatever of complexity, short of an infinite degree of complexity. That is, we must be able to construct and to evaluate (at least in principle if not in practice) any truth-functional compound sentence having any finite number of truth-functional operators. The Rules for Well-formedness allow us to construct sentences of any degree of complexity whatever. But how shall we evaluate intricate compound sentences? How might we evaluate a sentence such as " ~ ^ A" in which there are two operators; and how might we evaluate a still more complicated sentence such as "(AD ~ B) • (^ AoB)" in which there are five operators? To answer this question we shall have to see how the truth-tables of the previous section might be used, and this requires that we make a distinction between sentence-variables and sentence-constants. The "P"s and "Q"s which were featured in our truth-tables for negation, conjunction, disjunction, material conditionality, and material bi-conditionality, as arguments of the operators, " • ", "V" , " D " and " = " respectively, were sentence-variables. They stood indiscriminately for any proposition-expressing sentences whatever. But in addition to these kinds of symbols, we shall also want our conceptual notation to contain symbols which stand for specific sentences, and not — as variables do — for sentences in general. These symbols we shall call sentential-constants since they have a constant, fixed, or specific interpretation. We shall use capital letters from the beginning of the English alphabet — "A", "B", "C", "D", etc. — as our symbols for sentential-constants, and will reserve capital letters from the end of the alphabet — "P" through "Z" — as our symbols for sentential-variables.10 Finally we add that any wff containing a sentential-variable is to be called a 10. All capital letters of the English alphabet are to be considered wfFs, and hence the rules of the construction of wffs containing sentential-constants are just those already given. Sentence-form, while any wff containing only sentential-constants or containing only sentential-constants and sentence-forming operators, is to be called (simply) a sentence. To see how we might use the truth-tables of the previous section to evaluate truth-functional compound sentences containing any number of operators, we must view the sentential-constants in sentences as substitution-instances of the sentential-variables (i.e., the "P"s and "Q"s) featured on those tables. If the truth-values of the propositions expressed by the sentential-constants in a truth-functional sentence are given, then — by referring to the truth-tables for the various truth-functional operators — we may evaluate the whole sentence by means of a step-by-step procedure beginning with the simplest sentential components of that sentence, evaluating then the next more complex components of that sentence, repeating the procedure — evaluating ever more complex components — until the entire sentence has been evaluated.

A note on two senses of "determined" We have seen that each of the sentential operators "it is not the case that", "and", "or", "if then", and "if and only if admits of truth-functional uses — uses in which each generates compound sentences out of simpler ones in such a way that the truth-values of the propositions expressed by the compound sentences are determined by or are a function of the truth-values of propositions expressed by their simpler sentential components. In saying that the truth-values of the propositions expressed by truth-functional sentences are thus determined, we are, of course, making a purely logical point. We are saying, for instance, that what makes a proposition expressed by a compound sentence of the form " ^ P" true are just those conditions which account for the falsity of the proposition expressed by the simpler sentence "P", and that what makes a proposition expressed by a compound sentence of the form " ~ P" false are just those conditions which account for the truth of the proposition expressed by the simpler sentence "P"; we are saying that what makes a proposition expressed by a compound sentence of the form "PvQ" false are just those conditions which account for the falsity of both "P" and "Q"; and so on. The logical point we are making holds independently of whether anyone ever comes to know the truth-value of the propositions expressed by these compound sentences by coming to know the truth-values of the propositions expressed by their simpler sentential components. But there is another sense in which we can speak of the truth-values of propositions expressed by compound sentences in Truth-functional Propositional Logic being "determined". We may speak of the truth-values of these propositions being determined, in the sense of being ascertained, by us on the basis of our knowledge of the truth-values of the propositions expressed by their simpler sentential components. In saying that their truth-values may be thus determined we are, of course, making an epistemic point. The epistemic and logical points just made are, of course, connected. It is only insofar as the truth-values of the propositions expressed by compound sentences we are considering are, so to speak, logically determined by the truth-values of the propositions expressed by their simpler sentential components that we can determine, epidemically, what their truth-values are, given initial assignments of truth-values to the propositions expressed by their simpler sentential components. How these initial assignments are made is, of course, another story. Sometimes it is on the basis of experience: we know what value-assignment to make experientially. Sometimes it is on the basis of reason or analytical thinking: we know what value-assignment to make ratiocinative. And sometimes it is on the basis of mere supposition: we neither know experientially nor know ratiocinative what the truth-values of these simple sentential components happen to be, but merely assume or suppose them to be such and such or so and so. But in whatever way these initial value-assignments are made, it is clear that the consequential assignments that we make for the propositions expressed by compound sentences of which these simple sentences are the components can be made ratiocinative, and hence in a purely a priori way. Although the initial truth-value assignments may be made experientially or even empirically, the consequential assignments in a truth-functional propositional logic may be made a prior.

Check Your Progress 1

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1. Discuss the Truth Functional Operators.

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2. What is Evaluating Compound Sentences?

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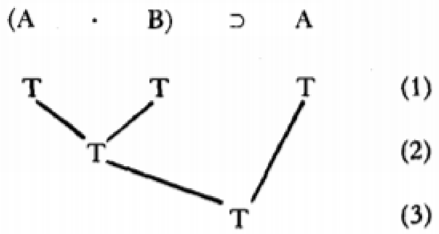
4.4 ELEMENTARY TRUTH-TABLE TECHNIQUES FOR REVEALING MODAL STATUS AND MODAL RELATIONS

Modal status So far we have seen how the method of evaluating a truth-functional sentence may serve to reveal the truth-value of the proposition expressed by that sentence. But the real importance for Truth-functional Propositional Logic of the technique of sentential evaluation lies elsewhere. The technique assumes far greater importance when it is extended to encompass not just an evaluation for one particular assignment of "T"s and "F"s to the sentential components in a complex sentence, but a series of evaluations for every possible assignment of "T"s and "F"s to the sentential components. As a matter of fact we have already done one such complete evaluation in the previous section when we evaluated the sentence ~A " first with "T" having been assigned to "A" and then subsequently with "F" having been assigned to "A". In that instance nothing particularly remarkable ensued. But there are other cases in which giving an exhaustive series of evaluations may serve to reveal various modal attributes of the propositions expressed. Perhaps this is best explained by beginning with an example. Suppose we start with the sentence (5.51) "(A-B)DA " Sentence (5.51) contains three sentential-constant tokens representing two sentential-constant types. In order to determine how many distinct assignments are possible for the sentential-constants in a sentence, we must count the types represented, not the number of tokens of those types occurring. In this instance the relevant number is two. The formula for ascertaining the number of distinct initial assignments, N, which can be made is simply, N = 2n , where "n" represents the total number of sentential-constant types represented. Thus there are 22 , i.e., four distinct initial assignments which might be made for (5.51). Rather than completing each evaluation in a tree-fashion as we did in the previous section, we will now write out each evaluation on the very same line as the one on which we make the initial assignment. In effect we simply compress the tree onto a single horizontal line. Thus instead of writing out the first evaluation of (5.51) in a tree-fashion such as



we will now write it out in this way:

(A	·	B)	⊃	A
T	T	T	T	T
(1)	(2)	(1)	(3)	(1)

12. Advanced truth-table techniques will be introduced in section 5.

Modal relations

By evaluating two truth-functional sentences together on one truth-table it is sometimes possible to ascertain mechanically the modal relation obtaining between the propositions those two sentences express. Suppose for example that we were to evaluate the following two sentences together: (5.54) "Today is Sunday and I slept late" and (5.55) "Today is Sunday or Monday." We would begin by translating these into the conceptual notation of Truth-functional Propositional Logic, e.g., (5.54a) "A-L " and (5.55a) "AVM " To evaluate both these wffs on a single truth-table we will require 23 rows.

4.5 THE LANGUAGE OF CLASSICAL SENTENTIAL LOGIC

Classical sentential logic (CSL) can be formulated in a ZOL, either in prefix (Polish) format or in infix (algebraic) format. In the former case, the formal language, o1, is specified as follows, using minimal notation.

(0) The vocabulary consists of the following: p, ÿ, N, K, D, C, B.
(a1) p is an atomic formula.
(a2) if s is an atomic formula, then so is sÿ.
(a3) nothing else is an atomic formula.
(f1) every atomic formula is a formula.
(f2) if s is a formula, then so is Ns.
(f3) if s1 and s2 are formulas, then so are:
Ks1s2
Ds1s2
Cs1s2
Bs1s2
(f4) nothing else is a formula.

The prefix connectives correspond to negation (N), conjunction (K), disjunction (D), conditional (C), and biconditional (B), respectively. On the other hand, the infix formulation of the language of CSL is given by formal language o2,
which is specified as follows.

(0) The vocabulary consists of the following: P, #, ;, &, Ú, ®, «, (,).
(a1) P is an atomic formula.
(a2) if s is an atomic formula, then so is sÿ.
(a3) nothing else is an atomic formula.
(f1) every atomic formula is a formula.
(f2) if s is a formula, then so is ;s.

(f3) if s1 and s2 are formulas, then so are:

(s1&s2)

(s1Ús2)

(s1®s2)

(s1«s2)

(f4) nothing else is a formula.

Notice that we have employed minimal notation in the metalanguage, rather than the grammatically more explicit quote/plus notation. In particular, rather than use quotes, we simply use the very same symbol in the metalanguage as the name of the symbol in the object language (one symbol, two meanings).

Also, rather than use '+', we adopt the implicit juxtaposition method for denoting complex expressions (strings) of the object language.
As a further notational simplification, from now on, we adopt the following official metalinguistic
definitions.
p0 =df p P0 =df P
p1 =df pÿ P1 =df Pÿ
p2 =df pÿÿ P2 =df Pÿÿ
etc. etc.
Notice that the numerical subscript is short for the number of occurrences of the sharp sign.

What's more, we will further adopt the following informal definitions of the customary atomic formulas of elementary logic.

P =df P0

Q =df P1

R =df P2

S =df P3

etc.[?]
[Alternatively, we could officially include all upper case Roman letters in our vocabulary, and declare that each of them is an atomic formula.]

4.6 TRUTH-VALUES

Ordinary sentential logic is not concerned with all sentences, but only declarative sentences, thus ignoring interrogative, imperative, exclamatory, and performative sentences. The simplest definition of a declarative sentence is that it is a sentence that is capable of being true or false. Basically, a declarative sentence is intended, when uttered, to declare something, which in turn is either true or false. It is furthermore customary to say that the sentence itself is true (resp., false) when what it declares is true (resp., false). Associated with the adjectives 'true' and 'false' are the abstract proper nouns 'True' and 'False', which refer to what are known as truth-values [more about reference later]. An analogy might be useful here. Consider the difference between the adjective 'blue' and the proper noun 'Blue', as used in the following two sentences my favorite shirt is blue my favorite color is Blue Observe that we capitalize the noun, in a Germanesque fashion, in order to further distinguish it from its corresponding adjective. On the other hand, we don't adopt Germanesque ontological sentiments; in particular, we do not automatically assume that there really is a thing (abstract or otherwise) to which the proper noun 'Blue' refers. Rather, we allow (but don't require) that the nominal use of 'blue' is merely a grammatical convenience. In order to hear the difference between the adjectival and nominal uses of 'blue', it is useful to see what happens when we invert the above sentences.

blue is my favorite shirt
Blue is my favorite color

The first one sounds funny (poetic, if you like); the second one sounds rather ordinary (prosaic, if you like).

Veterans of elementary logic can render the distinction in the starkest terms, by symbolizing the two sentences, as follows.

B[s(i)]

c(i) = b

B[a] : a is blue

s(a) : a’s the favorite shirt

c(a) : a’s the favorite color

i : I/me/my

b : Blue

Notice also that there is a natural semantic correspondence between the adjective ‘blue’ and the

noun ‘Blue’, given as follows.

object x is blue B[x]

if and only if «

the color of object x is Blue c(x) = b

Notice that this is not a logical truth (at least, not according to “standard” logic). On the other hand, it is

analytically true, which is to say it is true in virtue of the meanings of its terms.

Now back to truth-values. Just as there is a conceptual relation between ‘blue’ and ‘Blue’, there is

a relation between ‘true’ and ‘True’, and between ‘false’ and ‘False’. This is given as follows.

sentence Æ is true/false

if and only if

the truth-value of Æ is True/False

Check Your Progress 2

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1. What is Elementary Truth Table Techniques for Revealing Model Status and Model Relations?

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2. Discuss the Language of Classical Sentential Logic.

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3. What is Truth-Values?

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4.7 TRUTH-FUNCTIONS

The customary semantics for CSL employs the notion of truth-function, which is a function that takes truth-values as input and yields truth-values as output. Formally stated:

Df A truth-function is, by definition, an n-place function on {T,F}, for some number n.

For an account of functions, see the chapter on set theory. Basically, an n-place truth-function takes a ntuple of truth-values as input and delivers a truth-value as output; for example, a 2-place truth-function takes a 2-tuple (ordered pair) of truth-values and delivers a truth value as output.

The following are examples of 1-place, 2-place, and 3-place truth-functions.

(e1.1) f1(T) = T [f1 assigns T to T]

f1(F) = F [f1 assigns F to F]

(e1.2) f2(T) = F

f2(F) = T

(e1.3) f3(T) = T

f3(F) = T

(e1.4) f4(T) = F

f4(F) = F

(e2.1) g1(T,T) = T

g1(T,F) = F

g1(F,T) = F

g1(F,F) = F

(e2.2) g2(T,T) = F

g2(T,F) = T

g2(F,T) = T

g2(F,F) = T

(e3.1) h1(T,T,T) = T

h1(T,T,F) = T

h1(T,F,T) = F

h1(T,F,F) = F

h1(F,T,T) = T

h1(F,T,F) = F

h1(F,F,T) = T

h1(F,F,F) = F

(e3.2) h2(T,T,T) = T

h2(T,T,F) = F

h2(T,F,T) = F

h2(T,F,F) = T

h2(F,T,T) = T

h2(F,T,F) = F

h2(F,F,T) = T

h2(F,F,F) = F

How many truth-functions are there. Standard combinatorial reasoning yields the following finite results

(1)-(n). Set theory yields the general result.

(1) The number of 1-place truth-functions: 4

(2) The number of 2-place truth-functions: 16

(3) The number of 3-place truth-functions: 256

(4) The number of 4-place truth-functions: 64k [k = 1024]

(5) The number of 5-place truth-functions: 4096m [m = k2
]

(n) The number of n-place truth-functions: 2 exp (2 exp n)

(g) The number of truth-functions: infinitely-many

For example, in (2) there are four 2-tuples of truth-values; each one can be assigned T or F; so for each 2-
tuple there are 2 possible assignments. Accordingly, the total number of possible assignments is
2%2%2%2, which is 16. In the case of an n-place truth-function, there are 2n
(i.e., 2 exp n) different ntuples; for each n-tuple, there are 2 possible assignments, so the total number of possible assignments is 2
exp (2 exp n). The latter can be quite large.

4.8 TRUTH-FUNCTIONAL SEMANTICS FOR CSL

Intimately related to truth-functions are truth-functional connectives. A connective is not in and of it truth-functional, but is truth-functional only relative to semantics. Semantics for a formal language o provides, at the minimum, a set of admissible valuations on o, which are defined as follows. Df Let o be a language, and let S(o) be the set of sentences (formulas) of o. Then a valuation on o is any function from S(o) into {T,F}. A truth-value semantics on o is, by definition, any set of valuations on o. In this context, let us drop the prefix 'truth-value', and simply refer to a set V of valuations as semantics. We are now in a position to define truth-functionality

Df

Let c be an n-place connective in a prefix-formatted language o. Let V be a (truthvalue) semantics for o. Then c is truth-functional relative to V iff: there is an nplace truth-function, call it fc, such that, for every valuation u in V, for any formulas
f1,...,fn,
u(cf1...fn) = fc(u(f1),...,u(fn))

The basic idea is simple; a connective is truth-functional iff it corresponds to a truth-function. Insofar as connective c corresponds to truth-function fc, the truth-value of any c-formula is a function (specifically, fc) of the respective truth-values of its constituents.

If every connective of o is truth-functional relative to V, we say that V is a truth-functional semantics for o. This is made official in the following.

Df

Let o be a ZOL, and let V be a semantics for o. Then V is truth-functional iff every connective c of o is truth-functional relative to V.

The usual semantics for CSL is truth-functional. The following is a semi-formal definition of this semantics, for the prefix-formatted language o1.

Df

The usual semantics for CSL, in prefix-format, countenances as admissible all and only those valuations on o1 that satisfy the following restrictions.

(N) u(Na) = n(u(a))

(K) u(Kab) = k(u(a),u(b))

(D) u(Dab) = d(u(a),u(b))

(C) u(Cab) = c(u(a),u(b))

(B) u(Bab) = b(u(a),u(b))

Here, the truth-functions are defined as follows.

(n) n(T)=F; n(F)=T

(k) k(T,T)=T; k(T,F)=F; k(F,T)=F; k(F,F)=F

(d) d(T,T)=T; d(T,F)=T; d(F,T)=T; d(F,F)=F

(c) c(T,T)=T; c(T,F)=F; c(F,T)=T; d(F,F)=T

(b) b(T,T)=T; b(T,F)=F; b(F,T)=F; b(F,F)=T

The functions n,k,d,c,b are of course the familiar truth-functions associated, respectively, with negation, conjunction, disjunction, conditional, and biconditional. For example, the fact that k(T,T)=T amounts to the fact that the "conjunction" of T and T is T.

Whether we actually call the function k 'conjunction' depends upon how precise we wish to be. If we insist that conjunction is a connective, then the function k is not conjunction, since it is not a connective; rather, k is the truth-function that corresponds to conjunction. Of course, in intro logic, the connective and the truth-function were both called conjunction. [Intro students have enough trouble without having to worry about the distinction between (set theoretic) functions and (syntactic) functors.] On the other hand, it is convenient (if somewhat sloppy) to use the term 'conjunction' to refer to both the functor K and the function k. This allows us to describe the truth conditions for the functor K as follows. (t) the truth-value of the conjunction of two formulas is the conjunction of the truth-values of the two formulas. The latter statement can be made more precise, if we distinguish between syntactic conjunction and semantic conjunction, in which case (t) is rewritten as follows. (t*) the truth-value of the syntactic conjunction of two formulas is the semantic conjunction of the truth-values of the two formulas. Writing both "conjunctions" in infix notation, and using the same symbol '&' for both, we can re-write (t) as follows. (t**) u(a&b) = u(a) & u(b) Here, '&' is ambiguous: the first occurrence of '&' is the name of the ampersand symbol of the object language; the second occurrence is the name of the truth-function k, which is a set of ordered pairs. The difference between syntactic and semantic conjunction is striking; whereas (a&b) is a string consisting of '(' followed by a followed by '&' followed by b followed by ')', u(a)&u(b) is not a string but a truthvalue; for example, T&T is not a string consisting of T followed by & followed by T; T&T is just T [T&T=T]. The usual semantics for CSL is truth-functional. A simple example of a non-truth-functional semantics for o1 is easy to construct. (D) The semantics NTFS for o1 countenances as admissible exactly one valuation, namely w defined as follows. w(a)=T if u(a)=T for every u Î V(TFS); w(a)=F, otherwise. Here, V(TFS) is the set of admissible valuations of the usual truth-functional semantics, mentioned above. In other words, the valuation w assigns T to all tautologies of ordinary classical SL, but F to all nontautologies. To show that NTFS is not truth-functional, we need merely show that one connective is not truthfunctional. Consider negation; first, consider the formula Np; the input formula p is not a tautology of classical logic, so p is false in NTSF; similarly, the output formula Np is not a tautology, so Np is also false in NTSF. Input: false; output: false. Now, consider the negation NKpNp; the input formula KpNp is not a tautology of CL, so it is false in NTSF; on the other hand, NKpNp is a tautology of CL, so the output formula NKpNp is true in NTSF; input: false; output: true. Thus, relative to this semantics, the truth-value of a negation is not a function of the truth-values of its constituents.

4.9 EXPRESSIVE COMPLETENESS

Ordinary classical sentential logic (CSL) employs only five connectives, so the semantics of CSL only involves five truth-functions. Yet there are infinitely many truth-functions, and hence there are (in principle) infinitely many truth-functional connectives. As you already know from intro logic, many "nonstandard" truth-functional connectives can be paraphrased using "standard" truth-functional connectives. For example, 'neither...nor' sentences can be paraphrased using 'not' and 'and'; specifically, 'neither P nor Q' may be paraphrased as 'not-P and not-Q'. The obvious question that arises is whether every truth-functional connective (explicit or otherwise) can be paraphrased using standard connectives. If the answer is 'yes', then the standard connectives are expressively complete; if the answer is 'no', then the standard connectives are expressively incomplete. In formalizing this idea, we present the following definitions. Note carefully: In what follows, we presuppose a ZOL o and a semantics y for o; all definitions are relative to o and y. Df Two formulas a and b are said to be semantically equivalent iff u(a)=u(b) for every admissible valuation. Df Let ¶ be a collection of connectives, and let c be a connective. Then c is expressible in terms of ¶ iff every formula involving c is semantically equivalent to a formula involving just the connectives in ¶.

Finally, we observe that the connectives;, &, Ú can all be expressed in terms of a single connective "nor", which corresponds to 'neither...nor' (exercise). Similarly, they are expressible in terms of "nand", which corresponds to 'not both...and...'. Given the earlier theorem, it follows that every truth function is expressible in terms of a single truth-functional connective.

Check Your Progress 3

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

4. What is Truth-Functions?
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5. What is Truth-Functional Semantics for CSL?
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6. What is Expressive Completeness?
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4.10 LET US SUM UP

In presenting logic, the customary procedure involves four steps. In this unit we discussed:

(1) specify the syntax of the underlying formal language, o, over which the logic is defined;

(2) specify the semantics for o, in virtue of which semantic entailment is defined;

(3) specify a deductive system for o, in virtue of which deductive entailment is defined;

(4) show that semantic entailment and deductive entailment are mutually consistent.

In the present unit, we discuss steps 1 and 2 for classical sentential logic.

4.11 KEY WORDS

Define function; define truth-function; give examples from English of a 1-place, a 2-place, and a 3-place, truth-functional connective. In each case, write down the corresponding truth function in 'j(a)=v' notation.

Disjunctive Normal Form: Convert each of the following formulas into canonical disjunctive normal form; in other words, first construct the associated n-place truth function, then write down the DNF formula that yields this truth function.

QUESTIONS FOR REVIEW

1. Discuss the Truth Functional Operators.
2. What is Evaluating Compound Sentences?
3. What is Elementary Truth Table Techniques for Revealing Model Status and Model Relations?
4. Discuss the Language of Classical Sentential Logic.
5. What is Truth-Values?
6. What is Truth-Functions?
7. What is Truth-Functional Semantics for CSL?
8. What is Expressive Completeness?

4.13 SUGGESTED READINGS AND REFERENCES

- Roy T. Cook (2009). A Dictionary of Philosophical Logic, p. 294: Truth Function. Edinburgh University Press.

- Roy T. Cook (2009). A Dictionary of Philosophical Logic, p. 295: Truth Functional. Edinburgh University Press.
- Internet Encyclopedia of Philosophy: Propositional Logic, by Kevin C. Klement
- Roy T. Cook (2009). A Dictionary of Philosophical Logic, p. 47: Classical Logic. Edinburgh University Press.
- Wernick, William (1942) "Complete Sets of Logical Functions," Transactions of the American Mathematical Society 51: 117–32.

4.14 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress 1

1. See Section 4.2
2. See Section 4.3

Check Your Progress 2

1. See Section 4.4
2. See Section 4.5
3. See Section 4.6

Check Your Progress 3

1. See Section 4.7
2. See Section 4.8
3. See Section 4.9

UNIT 5: TRUTH - FUNCTIONAL FORMS

STRUCTURE

5.0 OBJECTIVES

The aim of this unit is to introduce you to the concept of equivalence through two means; truth table method and stroke and dagger function and contradiction through truth-table means. Though what you learn in this unit is much limited in terms of content, it forms the foundation of future learning. Hence this unit should prepare you to grasp the essence of the next block. After you are thorough with this unit you should be in a position to:

- To construct truth-tables for statements.
- To identify propositions having different form but same content.
- To reduce all verbal expression to non-verbal forms.

Notes

- To discover that verbal form is more complex and not necessarily useful when compared with symbolic form, which is simpler and more useful in our logical enterprise.

5.1 INTRODUCTION

In previous units the logical forms of specific statements expressed in English were discussed in aid of reasoning about their material truth values. Here, and in subsequent chapters there is an important shift in perspective. From now on, emphasis is on logical forms themselves, rather than specific statements which have those forms. The reason for this is that much of what we know about correct reasoning depends on forms of statements rather than material truth. After studying this chapter you should be able to:

1. Describe and use truth functional forms.

2. Describe and give examples of interpretations for truth functional forms.

3. Describe the conditions under which truth functional forms are said to be true or false for a given interpretation.

4. Distinguish among truth functionally true, false, and contingent forms.

5. Use properties of truth functional forms to help determine the logical status of statements expressed in English.

6. Use logical properties of forms to simplify statements and conditions.

In Unit 5 we learnt that in our study of symbolic logic we replace propositions by variables. These variables may be called prepositional variables because they signify indifferently any statement. Therefore whenever a propositional variable is assigned any truth-value, then the same truth-value has to be assigned to any proposition signified by the respective variable. We also learnt that sentential connectives help us to obtain compound propositions. While statements are variables, various connectives like 'not', 'if…then', etc., which produce compound propositions, are logical constants. A study of symbolic logic starts with what is known as, 'calculus of propositions or propositional calculus'. There are different forms of truth-function, which constitute propositional calculus with which we have to familiarize. In other words, various relations between propositions require to be studied. It is good to recapitulate what was discussed under compound statements. There are five kinds of compound propositions: implicative, conjunctive, disjunctive, negation and biconditional; each one defined by a definite form. An important aspect, which follows this discussion, is 'two kinds of relation which exist between these forms'. Contradiction and logical equivalence (equivalence in brief) are these forms with which we are concerned. The beginning of this study marks the beginning of the study of symbolic logic. Let us make a beginning with implication.

As noted earlier, an argument is valid or invalid purely in virtue of its form. The form of an argument is a function of the arrangement of the terms in the argument, where the logical terms play a primary role. However, as noted earlier, what counts as a logical term, as opposed to a descriptive term, is not absolute. Rather, it depends upon the level of logical analysis we are pursuing. In the previous chapter we briefly examined one level of logical analysis, the level of syllogistic logic. In syllogistic logic, the logical terms include 'all', 'some', 'no', 'are', and 'not', and the descriptive terms are all expressions that denote classes. In the next few chapters, we examine a different branch of logic, which represents a different level of logical analysis; specifically, we examine sentential logic (also called propositional logic and statement logic). In sentential logic, the logical terms are truth-functional statement connectives, and nothing else.

In the previous section, we examined the general class of (statement) connectives. At the level we wish to pursue, sentential logic is not concerned with all connectives, but only special ones – namely, the truth-functional connectives. Recall that a statement is a sentence that, when uttered, is either true or false. In logic it is customary to refer to truth and falsity as truth values, which are respectively abbreviated T and F. Furthermore, if a statement is true, then we say its truth value is T, and if a statement.

The **truth value** of a true statement is **T**.

The **truth value** of a false statement is **F**.

person. Just as we can say that the weight of John is 150 pounds, we can say that the truth value of ‘it is raining’ is T. Also, John's weight can vary from day to day; one day it might be 150 pounds; another day it might be 152 pounds. Similarly, for some statements at least, such as ‘it is raining’, the truth value can vary from occasion to occasion. On one occasion, the truth value of ‘it is raining’ might be T; on another occasion, it might be F. The difference between weight and truth-value is quantitative: whereas weight can take infinitely many values (the positive real numbers), truth value can only take two values, T and F.

The analogy continues. Just as we can apply functions to numbers (addition, subtraction, exponentiation, etc.), we can apply functions to truth values. Whereas the former are numerical functions, the latter are truth-functions. In the case of a numerical function, like addition, the input are numbers, and so is the output. For example, if we input the numbers 2 and 3, then the output is 5. If we want to learn the addition function, we have to learn what the output number is for any two input numbers. Usually we learn a tiny fragment of this in elementary school when we learn the addition tables. The addition tables tabulate the output of the addition function for a few select inputs, and we learn it primarily by rote. Truth-functions do not take numbers as input, nor do they produce numbers as output. Rather, truth-functions take truth values as input, and they produce truth values as output. Since there are only two truth values (compared with infinitely many numbers), learning a truth-function is considerably simpler than learning a numerical function. Just as there are two ways to learn, and to remember, the addition tables, there are two ways to learn truth-function tables. On the one hand, you can simply memorize it (two plus two is four, two plus three is five, etc.) On the other hand, you can master the underlying concept (what are you doing when you add two numbers together?) The best way is probably a combination of these two techniques.

5.2 IMPLICATION AND ITS EQUIVALENT FORMS

Let p stands for 'there is increase in supply' and q stands for 'the prices will fall'. Then, as we know already, the statement, 'if there is increase in supply, then the price will fall' is an implication (material implication to be precise) in a standard form. Our task is to derive its various equivalent forms and contradiction. As usual, we shall construct truth-table and then go to verbal form:

Table: 1

					Implication	Disjunction	Negation	
	p	q	⌐p	⌐q	p=>q	⌐p v q	⌐ (p Λ ⌐q)	
1	1	1	0	0	1	1	1	0
2	1	0	0	1	0	0	0	1
3	0	1	1	0	1	1	1	0
4	0	0	1	1	1	1	1	0

Under negation there are two columns which reflect truth-values. It must be remembered that the last but one column stands for equivalence relation. Therefore care should be taken to write the truth-value of negation exactly under the negation sign.The advantage of truth-value method is obvious. The equivalence relation, which exists between implication and disjunction, is selfexplanatory. However, relation with negation requires some clarification. There are two columns under negation, which reflect truth-values. Suppose that we ignore negation

Notes

sign and corresponding truth-values and consider the last column then we are not considering negation but conjunction. The last column is the same as the following one:

p Λ ⌐q
1 0
2 1
3 0
4 0

However, the required form is not conjunction but negation. The truth-value of negation, of course, truth-functionally depends upon the truth-value of conjunction form. Therefore while selecting the column, which corresponds to negation form, we should exercise a little caution. Now we shall consider the verbal form of logical equivalence. Suppose that the given proposition is as follows:

1) ‘If there is increase in supply, then the prices will fall’. The components of this
proposition and their symbols are as follows.
a). There is increase in supply. p
b). The prices will fall. q
The form of given proposition is as follows:
p => q -- (1)

If, instead of considering the form of any proposition, we symbolize propositions themselves, then we shall choose the first letter of the first term (in which case we ignore article, verb, etc.). In such a case we have to use upper-case letters. Then (1) is replaced by: I=>P. For some time let us use both the form of the proposition and symbols of given propositions.

Implication Disjunction Negation
a) p => q ≡ ⌐ p v q ≡ ⌐ (p Λ ⌐q)
b) I => P ≡ ⌐ I v P ≡ ⌐ (I Λ ⌐P)

Now we have to consider the negation of (a) and (b) mentioned above.

a') "There is no increase in supply" or

"It is not the case that there is increase in supply": ⌐ p/⌐I

b') "The prices will not fall" or

"It is not the case that the prices will fall": ⌐q /⌐ p

Disjunction: It is not the case that there is increase in supply or the prices will fall:

⌐ p v q / ⌐I v P

Negation: It is not the case that both there is increase in supply and the prices do not fall:

⌐ (p Λ ⌐q)/⌐ (I Λ ⌐P)

When negation is expressed in words, it is very important to observe that after 'it is not the case that' the word 'both' should invariably be used. Otherwise a mistake will be made. In fact, the word 'both' stands for the verbal expression of parentheses. Implication has one more equivalent form called contraposition. Its structure is as follows:

Table: 2

Implication Contraposition

p q ⌐ p ⌐ q p => q ⌐ q=> ⌐ p

4

1 1 1 0 0 1 1

2 1 0 0 1 0 0

3 0 1 1 0 1 1

4 0 0 1 1 1 1

Since sentential connective remains the same, the type proposition also remains the same. Hence its use is somewhat limited to the test of arguments.

5.3 DISJUNCTION AND ITS EQUIVALENT FORMS

If implication has equivalent disjunctive form, the converse also should hold good. The component proposition, 'there is increase in supply' and 'the prices will fall' are connected by the connective 'OR' and we obtain compound proposition as follows: 'There is increase in supply or the prices will fall'. Let us construct the truth table to be followed by verbal form.

Table: 3

	p	q	⌐p	⌐q	Disjunction p ∨ q	Implication ⌐p => q	Negation ⌐(⌐p Λ ⌐q)	
1	1	1	0	0	1	1	1	0
2	1	0	0	1	1	1	1	0
3	0	1	1	0	1	1	1	0
4	0	0	1	1	0	0	0	1

There is no difference in explanation for negation compared with implication. However, we shall repeat only truth-table form in order to eliminate any iota of doubt, if any. Accordingly, rewrite the truth-value of the last column.

⌐ p Λ ⌐q

1. 0
2. 0
3. 0
4. 1

Evidently, what we have here is only conjunction, but what we want is negation. Therefore the set of relevant truth-values belong to the last but one column, which is truth-functionally dependent upon those of the last column.

Let us switch over to verbal form and begin from disjunction.

2. There is increase in supply or the price will fall.

We shall rewrite the components and then append their negations.

a) There is increase in supply. p/I

b) The prices will fall. q/P

5

-a) There is no increase in supply or it is not the case that there is increase in supply.

¬p/¬I

-b) The prices will not fall. ¬q/¬P

Implication: If it is not the case that there is increase in supply, then the prices will fall.

¬ p => q or ¬I => P

Negation: It is not the case that both there is no increase in supply and the prices will not

fall. ¬ (¬ p Λ ¬ q) or ¬ (¬I Λ ¬P)

As in the case of implication, in this case also:

Disjunction Implication Negation

p v q ≡ ¬ p => q ≡ ¬ (¬ p Λ ¬ q)

Unlike implication, disjunction allows simple transposition of disjunctions. p v q ≡ q v p. In this case also transposition has limited application in the test of arguments. The rule which governs such simple transposition is known as rule of commutation. Therefore when we construct disjunctive syllogism, we are free to choose any component. The relation between (p v q) and ¬ (¬ p Λ ¬ q) is explained by what is known as de Morgan's law. It says that equivalence of disjunction consists in the negation of the conjunction of the negation of components. It is very important to understand this law completely and clearly. Here negation and conjunction are algebraic functions. Conjunction is equivalent to multiplication. We know that in algebra parenthesis also is equivalent to multiplication. Therefore negation within parentheses goes and negation outside parentheses remains. It shows that it is inadmissible to cancel three negation signs. To put it symbolically, ¬ (¬ p Λ ¬ q) ≠ (p Λ q). The method of testing this inequality is very simple.

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Table: 4

					Negation		Conjunction
	p	q	¬ p	¬ q	¬(¬ p ∧ ¬q)		p ∧ q
1	1	1	0	0	1	0	1
2	1	0	0	1	1	0	0
3	0	1	1	0	1	0	0
4	0	0	1	1	0	1	0

Since the truth-value of these expressions is not the same in all instances, they are not identical.

Check Your Progress 1

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1. Discuss the Implication and Its Equivalent Forms.

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2. Describe Disjunction and Its Equivalent Form.

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5.4 NEGATION AND ITS EQUIVALENT FORMS

The equivalent forms of negation take implication and disjunction forms when suitably translated. We have two components, ‘there is increase in supply’ and ‘the prices will fall’. When connected by ‘not’ we obtain,

3) It is not the case that both there is increase in supply and the prices will fall.

Let us rewrite the components and append their negations.

a) There is increase in supply. p/I

b) The prices will fall. q/P

⌐ a) There is no increase in supply or it is not the case that there is increase in supply.

⌐ p/⌐I

⌐ b) The prices will not fall or It is not the case that the prices will fall.

⌐q/⌐P

Now construct the truth-table for equivalent forms.

Table: 5

	P	q	⌐p	⌐q	Negation ⌐	(p Λ q)	Implication p =>⌐ q	Disjunction ⌐ p v ⌐ q
1	1	1	0	0	0	1	0	0
2	1	0	0	1	1	0	1	1
3	0	1	1	0	1	0	1	1
4	0	0	1	1	1	0	1	1

The verbal forms of relations are as follows: Implication: If there is increase in supply, then the prices will not fall. p => ⌐q / I => ⌐ P Disjunction: There is no increase in supply or the prices will not fall: ⌐ p v ⌐ q / ⌐I v ⌐P For negation also we do not consider transposition of components because it does not have any special significance. If the equivalent negation form of disjunction is given by de Morgan's law, then converse also naturally holds good. What is negated is negation of 'Conjunction'. That is, {⌐ (p Λ q)} is negated. Therefore negation sign goes. Conjunction is replaced by disjunction and components are replaced by their negations. Hence, we get disjunction, which is equivalent to negation. Before we pass on to check contradiction, it is good to challenge our own choice. Let us start with implication. How can we assert that only ⌐p v q is equivalent to p=>q? Why cannot we say that p v ⌐q is also equivalent? It is nearly impossible to give explanation in verbal form, as to how ⌐ p v q is equivalent to p => q, but not p v ⌐q. If

Notes

we compare the truth-values of ⌐ p v q and p v ⌐q with p => q, then the solution becomes clear.

Table: 6

	p	q	⌐p	⌐q	p=>q	⌐p v q	p v ⌐q	⌐p v ⌐q
1	1	1	0	0	1	1	1	0
2	1	0	0	1	0	0	1	1
3	0	1	1	0	1	1	0	1
4	0	0	1	1	1	1	1	1

⌐ p v ⌐ q is added only to reinforce our position. An equivalent expression must be true in only those instances in which the original expression is true (and in all such instances) and it must be false in only those instances in which the original expression is false (and in all such instances). According to this criterion, only ⌐ p v q is equivalent disjunctive form to the original implication. The students are advised to test all other cases, like disjunctive proposition, using truth-table method to conclude that other than those mentioned are not equivalent to the original expression. At this stage, it should become clear that use of verbal expressions to determine their equivalent forms renders the task an uphill task and sometimes practically impossible. It is left to the students to verify the last statement which he can do by considering fairly a complex statement. Now construct the scheme of equivalent expressions with truth- table.

Table: 7

	p	q	⌐p	⌐q		Implication	Disjunction	Negation
1	1	1	0	0	Implication: p => q	⌐ q => ⌐ p	⌐ p v q	⌐(p Λ ⌐q)
2	1	0	0	1	Disjunction p v q	⌐ p => q	(q v p)	⌐p Λ ⌐q
3	0	1	1	0	Negation ⌐ (p Λ q)	p=> ⌐q	⌐ p v ⌐q	⌐ (q Λ p)

It must be noted that while implication does not take equivalent converse form, disjunction and negation take.

5.5 CONJUNCTION AND BICONDITION

It is quite interesting to note that conjunction and bicondition do not have equivalent forms. Truth- table again comes to our rescue. It is sufficient if we consider any one-form, say, implication. If one equivalent form is absent, it is imperative that other forms are also absent

Table: 8

	p	q	¬p	¬q	p Λ q	p=>q	¬p=>q	p=>¬q	¬p=>¬q
1	1	1	0	0	1	1	1	0	1
2	1	0	0	1	0	0	1	1	1
3	0	1	1	0	0	1	1	1	0
4	0	0	1	1	0	1	0	1	1

Except that truth-values of conjunction do not tally with any possible arrangement in implication form, no other explanation is conceivable for the absence of equivalent forms to conjunction [The students are advised to test other forms to convince themselves]. Biconditional proposition also does not have any equivalent form. The reason is very simple. Biconditional is, in reality, conjunction only and both the conjuncts are implicative. First we shall know why it is regarded as conjunction.

Table: 9

					1	2	3	4
	p	q	¬p	¬q	p<=>q	(p=>q)	Λ	(q=>p)
1	1	1	0	0	1	1	1	1
2	1	0	0	1	0	0	0	1
3	0	1	1	0	0	1	0	0
4	0	0	1	1	1	1	1	1

The method of computing is as follows; first, we shall compute the truth-values of implication (p => q) and then we will compute the truth-values of q => p. These two sets of truth-values together determine the truth-value of conjunction. When we compare columns 1 and 3, we will come to know that these two expressions have identical truth-values in all instances. It shows that bicondition is also a conjunctive proposition where the conjuncts themselves are compound propositions. Therefore what applies to conjunction naturally, applies to bicondition also.

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Check Your Progress 2

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1. What is Negation and Its Equivalent Forms?

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2. What is Conjunction and Bicondition?

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5.6 FORM OF CONTRADICTION

When arguments are to be tested, quite frequently, we look for contradiction. Therefore it is necessary that we should know the contradiction of compound propositions so that with ease we can detect contradiction in arguments. The rule of contradiction is as follows: Whenever p is true its contradiction is false and whenever p is false its contradiction is true. That is to say contradiction and negation are same. The truth-table for contradiction is as follows.

Table: 10

	p	q	¬p	¬q	Implication p=>q	Contradiction p Λ ¬q
1	1	1	0	0	1	0
2	1	0	0	1	0	1
3	0	1	1	0	1	0
4	0	0	1	1	1	0

It is not difficult to express or understand contradiction in verbal form. We shall consider the components and their negation mentioned earlier.

Implication: If there is increase in supply, then the prices will fall.

a) There is increase in supply: p/I

b) The prices will fall: q/P

a') There is no increase in supply: ⌐ p /⌐ I

b') The prices will not fall ⌐ q /⌐ P

Contradiction: There is increase in supply and the prices will not fall:

p Λ ⌐q or I Λ ⌐P

As we challenged earlier conclusion, we shall again challenge this conclusion also. How can we say that p Λ ⌐q is the only contradiction? How do we know that this is the only form of contradiction permissible? Contradiction, in this case, does not have equivalent relation because p Λ ⌐q is a conjunction and conjunction does not have equivalent forms. As a rule, for any given proposition there is only one form of contradiction. We shall consider one disjunction form:

Table: 11

	p	q	⌐p	⌐q	Implication p=>q	Disjunction p v ⌐q
1	1	1	0	0	1	1
2	1	0	0	1	0	1
3	0	1	1	0	1	0
4	0	0	1	1	1	1

In order to test the conclusion, we effected only one change; we replaced conjunction by disjunction. In first and fourth instances, we notice that the truth-value remained the same whereas it should have been different. Therefore p v ⌐q is not a contradiction of implication. Contradiction of disjunction is, again, determined in accordance with de Morgan's law; replace disjunction by conjunction and disjuncts by their negations. Therefore the removal of negation prefixed to equivalent form of disjunction results in contradiction. The truth table is as follows:

Notes

Table: 12

	p	q	¬p	¬q	Disjunction p v q	Contradiction ¬p Λ ¬q
1	1	1	0	0	1	0
2	1	0	0	1	1	0
3	0	1	1	0	1	0
4	0	0	1	1	0	1

The verbal form is as follows:

Disjunction: There is increase in supply or the prices will fall: p v q / I v P

Contradiction: There is no increase in supply and the prices will not fall: ¬p Λ ¬q / ¬I Λ ¬P

[In this case also contradiction does not have equivalent forms. If the student wishes to test other alternatives, he or she can follow the method suggested earlier.] Contradiction of conjunction also is determined in accordance with de Morgan's law; replace conjunction by disjunction and the conjuncts by their contradictions.

Table: 13

	p	q	¬p	¬q	Conjunction p Λ q	Contradiction ¬p v ¬q
1	1	1	0	0	1	0
2	1	0	0	1	0	1
3	0	1	1	0	0	1
4	0	0	1	1	0	1

The verbal form is as follows: Conjunction: There is increase in supply and the prices will fall: p Λ q / I Λ P Contradiction: There is no increase in supply or the prices will not fall: ¬ p v ¬ q / ¬I v ¬P Since contradiction is in disjunctive form it has equivalent implicative form. I => ¬P, Ooviously, is its equivalent form. The contradiction of biconditional proposition is indirectly found and it is in accordance with de Morgan's law since its conjunctive feature is only concealed (i.e., the biconditional is a conjunction of two conditionals, as we see under 2,3,4 in the table) . Let us start with truth-table. [The verbal form is left out so that the student can attend the same.]

Table: 14 A B

							Contradiction
					1	2 3 4	5 6 7
	p	q	¬p	¬q	p<=>q	(p=>q)Λ(q=>p)	(pΛ¬q) v (qΛ¬p)
1	1	1	0	0	1	1 1 1	0 0 0
2	1	0	0	1	0	0 0 1	1 1 0
3	0	1	1	0	0	1 0 0	0 1 1
4	0	0	1	1	1	1 1 1	0 0 0

Compare columns 3 and 6. It becomes clear that A and B are contradictories.

For the sake of clarity let us consider contradiction in more than one step.

Step: 1 Contradiction components

Given expression Contradiction

a) p=>q p Λ ¬ q

b) q=>p q Λ ¬ p

Replace given expression by their contradictions, we obtain: (pΛ¬q)Λ(qΛ¬ p)

Step: 2 In Step1, we apply one aspect of de Morgan’s law, i.e., replacing conjunct by their

negation. In step 2 we apply second aspect of de Morgan’s law; i.e., replace conjunction

by disjunction. We get: (pΛ¬q) V (qΛ¬p)

We are only required to compare columns 3 and 6 to assure ourselves that the chosen and tested form is the contradiction of the original expression. We shall tabulate the results and at this stage we can omit the basic columns, i.e., truth-values of p, q, ¬ p & ¬q since we are familiar with the process involved.

Contradiction Form

a) Implication (p=>q) p Λ ¬q

b) Disjunction (p v q) ¬ p Λ ¬q

c) Conjunction (p Λ q) ¬ p v ¬q

d) Bicondition (p<=>q) (p Λ ¬q) Λ (q Λ ¬p)

It may be noted that when we compute equivalent forms we can do away with implication. We are at liberty to retain disjunction or conjunction.

Notes

Only negation is constant. Since we can derive from negation and disjunction all other sentential connectives, these two one are called primitive connectives. (However, bicondition is an exception). Such a process results in a sort of simplification since the number of connectives we require comes down as a result of this process. In order to further reduce the number of connectives, a different technique was introduced. This is known as stroke and dagger operation.

5.7 THE STROKE FUNCTION

Though the stroke function was introduced by C.S. Peirce, it is better known as the Shefferfunction after H.M. Sheffer, a mathematician. This function has negative force. The stroke function also is called stroke operator. This is also a connective because its use determines the truth-value of compound proposition, given the truth-value of its components.The definition of this function can be attempted in this fashion: "When a stroke connects any two statements, then it has to be construed that at least one of them is false, if the function itself must be true." Suppose that p and q are statements forms. Then p | q means that either p is false or q is false when p | q is true. This definition does not rule out the possibility of both p and q being false. It can be depicted in the following manner:

Table: 15

	p	q	p | q
1	1	1	0
2	1	0	1
3	0	1	1
4	0	0	1

Accordingly, compound propositions can be expressed in stroke form in the following manner:

1). Negation:

Truth-table method Stroke Method

p ¬p p│p

1 0 0

0 1 1

2). Conjunction

Table: 16

Truth-table method **Stroke method**

					1	2 3 4
	p	q	¬p	¬q	p∧q	(p│q) │ (p│q)
1	1	1	0	0	1	0 1 0
2	1	0	0	1	0	1 0 1
3	0	1	1	0	0	1 0 1
4	0	1	1	1	0	1 0 1

This process needs some explanation and explanation is in terms of truth-value. Consider p│q and apply the definition of stroke function. p│q is false only when both p and q are true, i.e. in the first instance only. In all other instances, from Table 14, we understand that at least one of them is false. So the stroke function is true. Now consider columns 2 and 4. Only in the first instance '0' appears in these two columns and nowhere else '0' appears in columns 2 and 4. Therefore in accordance with the definition of stroke function column 3 takes the value 1 only in the first instance. When we compare column (1) and (3) we learn that there is agreement in terms of the truth-value in all the instances. Therefore (p Λ q) ≡ (p│q) | (p│q), i.e., they are logically equivalent.

Table 17

3). Disjunction:

Truth-table method Stroke method

					1	2 3 4
	p	q	⌐p	⌐q	p v q	(p│p) | (q│q)
1	1	1	0	0	1	0 1 0
2	1	0	0	1	1	0 1 1
3	0	1	1	0	1	1 1 0
4	0	0	1	1	0	1 0 1

Considering the fact that stroke function is somewhat subtle, explanation is desirable. Apply the definition of stroke function to p│p and q│q. p│p is true only in 3rd and 4th instances where p is false. According to the definition of stroke functions, stroke function is true only when at least one component is false. p│p is false 1st and 2nd instances when p is true. Similarly, q│q is true in 2nd and 4th instances when q is false. Now apply stroke function to column 3. It takes the value 1in the first three instances since 0 appears either in column 2 or column 4 in these instances. It can take the value '0' only in the fourth instance since only in this instance the columns 2 and 4 take the value1. When we compare columns (1) and (3) we learn that there is agreement in terms of the truth-value in all the instances. Therefore, (p v q) ≡ (p│p) | (q│q), i.e., they are logically equivalent.

3). Implication:

Table 18

Truth-table method Stroke method

					1	2 3 4
	p	q	⌐p	⌐q	p => q	p │ (q│q)
1	1	1	0	0	1	1 1 0
2	1	0	0	1	0	1 0 1
3	0	1	1	0	1	0 1 0
4	0	0	1	1	1	0 1 1

The truth-value, which appears in column 3 is truth-functionally dependent on truth-values, which appear in columns 2 and 4. Column 3 takes the value 1 in instances 1, 3 and 4.Since in these instances ‘0’ appears in one or the other column. Only in second instance column 3 takes the value 0 since columns 2 and 4 both take the value 1.This is in accordance with the definition of stroke function. Columns 1 and 3 agree in all the instances in terms of truth-value. Therefore, (p=>q) ≡ p | (q | q), they are logically equivalent.

5.8 THE DAGGER FUNCTION

The dagger version can be regarded as stronger variation of the stroke function. When a compound proposition is expressed in terms of stroke function, the rule is that at least one of the components must be false if the stroke function must be true, though the possibility of both being false to make stroke function true is allowed. However, in dagger function, both the components must be false to make it true. This statement is regarded as the definition of dagger function. Suppose p and q are statements forms, then, p↓q is true if and only if both p and q are false. Otherwise, it is false. Accordingly, compound propositions can be expressed in dagger form in the following manner.

Table: 19

	p	q	p ↓ q
1	1	1	0
2	1	0	0
3	0	1	0
4	0	0	1

Explanation is left out so that the student can attempt the same.

4). Implication:

(p => q) ≡ {(p↓p)↓q}↓{(p↓p)↓q}

{

Explanation and truth-table are left out so that the student can attempt the same.

Biconditinal proposition can be expressed neither in stroke form nor in dagger form. So it does not have any form of equivalence. Negation of conjunction also does not have equivalence in these forms.

Check Your Progress 3

Note: a) Use the space provided for your answer.
b) Check your answers with those provided at the end of the unit.

3. Discuss the Form of Contradiction.
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4. What is The Stroke Function?
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5. What is The Dagger Function?
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5.9 LET US SUM UP

Truth-function and variables are basic to propositional calculus. Symbolic logic begins with propositional calculus. Compound propositions are characterized by both variables and constants. Contradiction and equivalence are two important logical relations. While conjunction and bicondition do not have equivalent forms other compound propositions have equivalent forms. Equivalent forms eliminate all but two connectives, negation and disjunction, which are primitive connectives. Stroke and dagger operators reduce the number to one. Only bconditional remains unaffected.

We will discuss several examples of truth functions in the following sections. For the moment, let's look at the definition of a truth-functional connective. A statement connective is truth-functional if and only if the truth value of any compound statement obtained by applying that connective is a function of (is completely determined by) the individual truth values of the constituent statements that form the compound. This definition will be easier to comprehend after a few examples have been discussed. The basic idea is this: suppose we have a statement connective, call it ‘+’, and suppose we have any two statements, call them S1 and S2. Then we can form a compound, which is denoted S1+S2. Now, to say that the connective ‘+’ is truthfunctional is to say this: if we know the truth values of S1 and S2 individually, then we automatically know, or at least we can compute, the truth value of S1+S2. On the other hand, to say that the connective + is not truth-functional is to say this: merely knowing the truth values of S1 and S2 does not automatically tell us the truth value of S1+S2. An example of a connective that is not truth-functional is discussed later.

5.10 KEY WORDS

Operator: In symbolic logic ‘operator’ means a tool with the help of which an action is performed. Here the act consists in determining the truth-value of a compound proposition.

Constant: A constant is a quantity that does not change, over time or otherwise. It has a fixed value.

Variable: A variable is a symbol for which there many suitable substitutions.

5.11 QUESTIONS FOR REVIEW

1. Discuss the Implication and Its Equivalent Forms
2. Describe Disjunction and Its Equivalent Form.

3. What is Negation and Its Equivalent Forms?
4. What is Conjunction and Bicondition?
5. Discuss the Form of Contradiction
6. What is The Stroke Function?
7. What is The Dagger Function?

5.12 SUGGESTED READINGS AND REFERENCES

- Alexander, P. An Introduction to Logic, London: Unwin University, 1969.
- Basson, A.H. & O'connor, D.J. Introduction to Symbolic Logic. Calcutta: Oxford University Press, 1976.
- Copi, I.M. Introduction to Logic. New Delhi: Prentice Hall India, 9th Ed., 1995.
- Joseph, H.W.B. An Introduction to Logic. Oxford: 1906.
- Lewis, C.I. & Longford, C.H. Symbolic Logic. New York: Dover Pub. Inc.,1959.

5.13 ANSWERS TO CHECK YOUR PROGRESS

Check your Progress 1

1. See Section 5.2
2. See Section 5.3

Check your Progress

1. See Section 5.4
2. See Section 5.5

Check your Progress 3

1. See Section 5.6
2. See Section 5.7
3. See Section 5.8

UNIT 6: COMPOUND STATEMENTS AND THEIR TRUTH-VALUES

STRUCTURE

6.0 OBJECTIVES

After you clutch the contents of this unit you should be in a position to:

- To analyze any compound proposition to determine its truth-value.

- To realize that always symbolic representation of statements helps better understanding than verbal representation which is not only more complicated in structure but also ambiguous.

- To understand that a compound proposition may be highly complicated as far as structure is concerned, but it does not affect the technique of determining the truth-value.

- To determine the width of spectrum of compound proposition and simple form of compound from the complicated form of compound proposition.

6.1 INTRODUCTION

In this unit an attempt is being made to project the structure of and variety in proposition in a new perspective. Secondly, two shades of meaning of compound proposition will be distinguished in order to accommodate one type of statements, which looks like simple. A clear definition of truth-function is attempted by considering two parameters simultaneously.

A statement is either simple or compound. A simple statement encloses no other statement. A compound statement can enclose simple statements and other compound statements.

```
sequence_of_statements ::= statement {statement}

statement ::=
  {label} simple_statement | {label} compound_statement

simple_statement ::= null_statement
  | assignment_statement | procedure_call_statement
  | exit_statement       | return_statement
  | goto_statement       | entry_call_statement
  | delay_statement      | abort_statement
  | raise_statement      | code_statement

compound_statement ::=
    if_statement         | case_statement
  | loop_statement       | block_statement
  | accept_statement     | select_statement

label ::= <<label_simple_name>>

null_statement ::= null;
```

A statement is said to be labeled by the label name of any label of the statement. A label name, and similarly a loop or block name, is implicitly declared at the end of the declarative part of the innermost block statement, subprogram body, package body, task body, or generic body that encloses the labeled statement, the named loop statement, or the named block statement, as the case may be. For a block statement without a declarative part, an implicit declarative part (and preceding declare) is assumed.

The implicit declarations for different label names, loop names, and block names occur in the same order as the beginnings of the corresponding labeled statements, loop statements, and block statements. Distinct identifiers must be used for all label, loop, and block names that are implicitly declared within the body of a program unit, including within block statements enclosed by this body, but excluding within other enclosed program units (a program unit is either a subprogram, a package, a task unit, or a generic unit).

Execution of a null statement has no other effect than to pass to the next action.

The execution of a sequence of statements consists of the execution of the individual statements in succession until the sequence is completed, or a transfer of control takes place. A transfer of control is caused either by the execution of an exit, return, or goto statement; by the selection of a terminate alternative; by the raising of an exception; or (indirectly) by the execution of an abort statement.

Examples of labeled statements:

```
<<HERE>> <<ICI>> <<AQUI>> <<HIER>> null;

<<AFTER>> X := 1;
```

6.2 SIMPLE AND COMPOUND STATEMENTS

In this unit, we consider two kinds of statements; simple and compound. This kind of distinction is similar to grammatical distinction. However, there is a sharp difference. A compound statement in grammatical sense is independent of its components as far as its truth-value is concerned.

However, in logical sense the truth or falsity of compound proposition depends upon the truth or falsity of its components. Simple proposition does not need any definition. It consists of only one sentence in grammatical sense. Compound statement, on the other hand, consists of two or more than two 'statements'. The last word should be carefully observed. It just says 'statements'. In other words, the components of a compound statement may be simple or themselves compound. Though the distinction per se is too a simple, statements may be deceptive. Consider the following examples:

1 Grass is green.

2 Einstein is a physicist and Lorenz was his professor.

3 Descartes is a philosopher and mathematician.

It is easy to conclude that the first statement is simple and the second statement is compound. However, we should not be hasty in judging the third proposition. It only seems to be a simple proposition. In reality, it is a compound statement. It can be analysed as follows: Descartes is a philosopher and Descartes is a mathematician. In the language of predicate logic compound proposition can be understood as follows; if there are two predicates then there are two propositions. And if there are three predicates, then there are three propositions and so on.

6.3 SENTENTIAL CONNECTIVES

A compound proposition can be generated in several ways. Classical logic says that a proposition is generated when subject and predicate terms are conjoined by copula. Likewise, modern logic says that a compound proposition is generated when two or more than two propositions are conjoined by what is known as sentential connective. There are five types of sentential connectives and therefore, there are five types of compound statements. 'And', 'if…then', 'or', 'not' and 'if and only if' (iff) are the connectives used to conjoin the statements. While providing descriptive account, connectives are shown, initially, in upper case letters for the sake of clarity only. Further, all letters printed in lower case below statements symbolise respective statements.

I) AND: 'AND' is one type of sentential connective. When two propositions are connected by this connective, a compound proposition is generated. This type of compound proposition is known as 'CONJUNCTIVE' proposition or we simply say 'CONJUNCTION'. Consider first simple propositions: Water flows down hill. The sun is bright. It is very easy to form a conjunctive proposition; just place 'AND' between 'water flows downhill' and 'the sun is bright'. We get the statement Water flows downhill AND the sun is bright. p q When we are doing symbolic logic, we hardly construct statements with words. Nor do we use 'AND' while writing a conjunctive proposition. Otherwise, it ceases to be symbolic logic. This connective is symbolized in two ways. The old style is '.' And the present style is 'Λ'. We will follow the latter. Now we will symbolize the proposition: Water flows downhill: p The sun is bright: q The conjunction is as follows: (water flows downhill) and (the sun is bright). p Λ q p Λ q is the form of conjunction. When an argument is being tested propositions are symbolised in the following manner. p is replaced by W and q is replaced by S; therefore p Λ q is replaced by W Λ S. This change is useful when there are several statements. This particular classification applies equally to other compound propositions, which involve other sentential connectives.

II) IF….THEN: A compound proposition generated with this particular connective is known as 'IMPLICATIVE' proposition or simply 'IMPLICATION'. It is also called hypothetical. The latter, usage, however, is restricted only to classical logic. In order to obtain implicative proposition the first word 'if' is inserted in the very beginning of compound proposition; 'then' is inserted between two components. We will show the process of conjoining these statements with an example: ‘There is no end to political turmoil’; ‘Economic prosperity will be badly hit’. We obtain the following implicative proposition: ‘IF there is no end to political turmoil, THEN economic prosperity will be badly hit.’ We shall symbolize it as follows: 7 There is no end to political turmoil: p 8 Economic prosperity will be badly hit: q 9 If p, then q.; this is the form of implicative proposition. Replace the form by symbols for propositions. We get If T, then E. Now we will take second step. The connective ‘if..... then’ also is symbolized. Again there are two ways of symbolizing the same. ‘⊃’ and ‘=>’. We shall use only the latter; p => q. ‘⊃’, which is read horse shoe, is not used now to show implication because this symbol is used in set theory to show class inclusion. In order to avoid ambiguity and confusion we represent implication with the symbol =>.

III) III) OR: When ‘OR’ connects two propositions we obtain DISJUNCTIVE proposition or simply DISJUNCTION. Some authors like Cohen and Nagel preferred to call it ALTERNATIVE proposition or simply ALTERNATION. At the outset, we should distinguish two senses in which this connective is often used. One is called 4 ‘inclusive’ or and the second one is called ‘exclusive’ or. The process of obtaining disjunction is very simple. The connective ‘OR’ is placed between simple propositions. The resultant statement is a disjunctive one. Take these statements: 10 Reason is the true friend of mankind. p 11 Treason is the worst enemy of the state. q With these two statements we obtain the required disjunctive statement: 12 ‘Reason is the true friend of mankind OR Treason is the worst enemy of the state’. When it is symbolized, it becomes p or q. The connective ‘OR’ is symbolized by using the symbol ‘v’. This symbol is called Wedge. p or q becomes p v q. This particular statement is an example for ‘inclusive’ OR. It is called inclusive because the statement also includes third possibility. Accordingly, it can be further extended in the following manner: 13 ‘Reason is the true friend of mankind or treason is the worst enemy of the state' or both. The last word 'both' is the extended part of original compound statement. This is third possibility, which cannot be logically ruled out. If third possibility is admissible in any disjunctive proposition, then 'OR' becomes inclusive. There are cases when third possibility is not admissible. Consider these two statements: 14'Rich people are generous or greedy.' It does not admit further extension. It does not make sense to say that 15 'Rich people are generous or greedy or both generous and greedy.' Since the extended part is inadmissible in this example 'OR' is regarded as exclusive or. When disjunction consists of exclusive or, the proposition is symbolized as p v q At this juncture a clarification is necessary. When is 'OR' inclusive and when is it exclusive? There is no law of logic as such which stipulates the conditions under which 'OR' becomes inclusive and conditions under which 'OR' becomes exclusive. We have to depend upon the 'meaning' of certain terms employed in the construction of statements. Consider propositions 10 and 11. We admit that these two statements do not exclude each other based on what these statements 'really' mean. However the same is not the case with propositions 14. The terms 'greedy' and 'generous' mean so differently that they both 'cannot' be the attributes of the very same class or individual. In other words, if rich people are greedy surely some other class of people can be generous and vice versa. Hence meaning alone can be our guide in determining whether 'or' is inclusive or exclusive. Generally, disjunction is expressed in terms of 'EITHER ... OR'. There is no harm in omitting the former. Both usages are admissible.

IV) NOT: In modern logic, when the connective NOT is appended to the given propositions, it becomes a compound proposition. However, grammar does not allow it. Therefore we have to treat this as a special case within the structure of modern logic. We obtain 'NEGATION' when NOT is used.

Notes

This is another kind of compound proposition in strictly logical sense because the use of this word alters the truth-value of the given proposition. The connective NOT is appended to the given propositions in several ways. Negation may begin with expressions like “It is NOT the case that… … …” or “it is NOT true that… … …” Consider this example: 16 The sun rises in the east. - p Now this statement is negated and expressed in three different ways. 17 It is NOT the case that the sun rises in the east. - NOT p 17a It is NOT true that the sun rises in the east. - NOT p 17b The sun does NOT rise in the east. - NOT p It must be noted that all these three statements exactly mean the same and all of them negate the statement 16. Now we will symbolize the statement, using symbol for negation, ‘⌐’ 16 p 17 ⌐ p ‘Not’ was symbolized earlier in a different way. The symbol ‘~’ was used earlier to denote negation. This is read curl or tilde. Russell and others used this symbol. V IF AND ONLY IF: When this connective is used we obtain ‘BICONDITION’. We will insert this connective between two statements to obtain ‘BICONDITIONAL’ proposition. Consider these two examples: 18 Mr. A is a bachelor. - p 19 Mr. A is an unmarried male. - q Now connect 18 and 19 using the given connective. Mr. A is a bachelor IF AND ONLY IF Mr. A is an unmarried male. This connective is symbolized in this manner ‘<=>’. BICONDITIONAL proposition is represented as follows; p <=> q. Negation (⌐) and biconditional are (p <=>q) special kinds of compound proposition. This will become clear in the next section.

Check Your Progress 1

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1) Distinguish 'compound' in grammatical sense from 'compound' in logical sense.

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2) Bring out the difference and similarity with respect to copula and sentential connective.

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6.4 COMPOUND PROPOSITIONS AND THEIR TRUTH-VALUES

Classical logic stipulates that any proposition is either true or false. The truth-value of a true proposition is TRUE and the truth-value of a false proposition is FALSE. Truthvalue refers to the designating of a statement either as true or false. Likewise, any compound proposition is either true or false. There is a technique of determining the truth-value of compound proposition. In effect the truth-value of a compound proposition is a function of the truth-value of its constituent or component statements. Barring a few cases, which are exceptions, in all other cases this particular specification applies to compound proposition. Therefore it is very important to distinguish these two kinds of compound proposition. It is distinguished as follows: 'A compound proposition is said to be truth- functionally compound if and only if its truth-value is a function of the truth-value of its components'. In other words, truth-function is a compound statement whose truth-value is completely determined by the truth-values of its components. Logic which deals with truth-functional compound statements is called truth–functional logic: this is the part that we are presently studying. The construction of truth-table (which is the list that shows the various values a truthfunction may assume) is a technique adopted in order to determine the truth-value of compound propositions. It is interesting to learn that even when the propositions remain the same, different types of compound propositions exhibit different truth-values because sentential connectives change from one compound to another compound. This clearly shows that the sentential connective plays a crucial role in determining the truth-value of a compound proposition. Therefore the truth-value of a compound proposition is determined by the truth-values of components and also the sentential connective used. In order to drive home this paint, let us retain the same set of statements, which form parts of compound proposition, but at the same time obtain different results in terms of truthvalues by using different sentential connectives?

21 The stars are self-luminous. - p

22 Glass is fragile. - q

Let us construct truth-tables to determine the truth-values of compound propositions (As usual '1' stands for 'True' and '0' stands for 'False'). Generally, no justification for determination of truth-value is called for. They are to be treated as the truth-conditions of respective compound propositions.

I) IMPLICATION: An implicative proposition is false only under one circumstance, i.e., when the antecedent is true and the consequent is false. It means that false conclusion does not follow from true premise and under all other circumstances it is true. In the case of implication antecedent is the premise and consequent is the conclusion. Let us illustrate it in the form of a table.

Table1:

	p	q	p => q
1	1	1	1
2	1	0	0
3	0	1	1
4	0	0	1

From this table one aspect becomes clear; a false premise implies any conclusion (whether true or false). It also means that a true conclusion follows from any premise. This is admissible because there is no necessary relation between the premise and the conclusion as pointed out earlier. Implication as understood in logic is very different from common man's perception. This is exactly what Russell meant when he introduced the term 'material implication'. Let us consider implication in verbal form. The statement 'If the stars are self-luminous, then glass is fragile' is false only when it is true that the stars are self-luminous and it is not the case that glass is fragile; and under all other circumstances it is true. This entire expression is hidden in Table 1. It is anybody's guess that Table 1 is more intelligible and understood with less effort than verbal form.

II) CONJUNCTION: A Conjunction is true if and only if both the conjunctions are true; otherwise, it is false. Therefore, the truth-table for conjunction is as follows:

Table 2

	p	q	p Λ q
1	1	1	1
2	1	0	0
3	0	1	0
4	0	0	0

Conjunction corresponds to a familiar algebraic rule. When two positive numbers are added we will get sum. However, when a negative number is added to a positive number, we are only subtracting. And addition of two negative numbers also amounts to subtraction only. (-4+ (-4) = -8; and –8 < -4). Let us restate conjunction in verbal form: i) The stars are self-luminous: 1 ii) Glass is fragile: 1 Conjunction: 1 The Stars are self-luminous and glass is fragile. 1 2 The Stars are self-luminous and glass is not fragile: 0 3 The Stars are not self-luminous and glass is fragile: 0 4 The Stars are not self-luminous and glass not fragile: 0 I

III) DISJUNCTION: A disjunction is true when at least one of the disjuncts is true. The condition of its truthvalue can also be stated in this manner. A distinction is false if and only if both the disjuncts are false. Stated in this form, disjunction is just the inversion of conjunction. The truth-value for disjunction is as follows.

Table: 3

	p	q	p v q
1	1	1	1
2	1	0	1
3	0	1	1
4	0	0	0

At a later stage we will have an opportunity to understand the significance of the way in which the truth-value conditions of disjunction and conjunction differ. For the time being, let us consider the verbal form of disjunction.

i) The Stars are self-luminous. 1

ii) ii) Glass is fragile. 1

Disjunction:

1 The stars are self-luminous or glass is fragile. 1

2 The stars are self-luminous or glass is not fragile. 1

3 The stars are not self luminous or glass is fragile. 1

4 The stars are not self luminous or glass is not fragile. 0

IV) NEGATION: The simplest form of truth-functionally compound proposition is negation. In this case we have only two rows because there is only one proposition whereas in all other cases there are four rows because there are two propositions.

Table: 4

	p	⌐ p
1	1	0
2	0	1

If p stands for 'The stars are self-luminous', ⌐ p stands for 'The stars are not self luminous'. Therefore if 'it is true that the stars are self-luminous', then it is not true that the stars are not self-luminous'. And if it is not the case that the stars are self-luminous, then it is true that the stars are not self-luminous. Again, it is obvious that the verbal form is more complex than the truth-table. Since negation connects one proposition only, it is called unary whereas all other connectives are called binary since they connect two propositions.

V) BI-CONDITION: A biconditional proposition is true only when both the components have the same truthvalue. Otherwise, it is false. The truth-value of biconditional proposition is as follows:

Table: 5

	p	q	p <=>q
1	1	1	1
2	1	0	0
3	0	1	0
4	0	0	1

Now let us consider verbal form for bicondition. 'The stars are self-luminous if and only if glass is fragile' is true when 'it is the case that the stars are self-luminous' and also 'it 10 is the case that glass is fragile' or when 'it is not the case that the stars are self-luminous' and also it is not the case that glass is fragile'. Under remaining circumstances, it is false. In such cases the verbal form is as follows:

1 The stars are not self-luminous if and only if glass is fragile.

2 The stars are self-luminous if and only if glass is not fragile. Again, let it be made clear that whether we say it is not the case that ‘the stars are selfluminous’ or we say that ‘the stars are not self-luminous, there is no difference in intended meaning. Negation and bicondition are unique for different reasons. Negation is unique because, though in grammatical sense, the statement ‘the stars are not self-luminous’ is a simple statement, modern logic regards it as a compound statement only because its truth-value depends upon the inclusion or exclusion of the connective ‘not’. So what determines the compound nature of a proposition is not really the number of statements, but it is the truth-functional quality of proposition. In this connection it is worthwhile to refer to exceptions mentioned in the beginning of this section. While all truth- functional statements are compound, all compound statements are not truth-functional. In other words, in exceptional cases, the truth-value of components does not determine the truthvalue of ‘apparent’ compound propositions. Consider these propositions, which, obviously, have this form.

23. If there is rise in the temperature, then there is rise in mercury level.
24. If India has to win the cricket match, then the gods must be crazy.

(23) and (24) differ in structure, which we generally, do not notice easily. In order to clearly understand the difference, let us break (23) and (24) to get their respective components.

23a There is rise in temperature.

23b There is rise in mercury level.

24a India has to win.

24b The gods must be crazy.

(23a) and (23b) are true or false together. But the same cannot be said about (24a) and

(24b). They are, really, neither true nor false together. Therefore though (24) is a

compound sentence, it is not truth-functionally compound. Therefore what is grammatically a compound statement may not be truth-functionally compound and viceversa.

Bioconditional proposition is unique for another reason. Implication does not allow simple transposition of antecedent and consequent whereas biconditional proposition allows only simple transposition of components. Consider p => q and q => p respectively with the help of truth-table.

Table: 6

	p	q	p =>q	q => p
1	1	1	1	1
2	1	0	0	1
3	0	1	1	0
4	0	0	1	1

From rows (2) and (3) it becomes clear that (p =>q) ≠ (q =>p). This is because the truth of implication does not allow simple transposition. However, the case of biconditional proposition is different. We should remember that many disputes can be settled with the help of truth-table.

Table: 7

	p	q	p <=>q	q <=>p
1	1	1	1	1
2	1	0	0	0
3	0	1	0	0
4	0	0	1	1

From tables (6) and (7) it is clear that what allows or does not allow simple transposition is the truth-condition only. This particular characteristic can be brought out clearly only when bicondition is contrasted with implication. The role played by sentential connectives in determining the truth-value of compound propositions vis-a-vis the truth-value of the components themselves is better understood when we compare the truth-table of all compound propositions. However, negation is not required for this purpose, since it does not have components.

Table: 8

	p	q	p =>q	p v q	p ∧ q	p<=>q
1	1	1	1	1	1	1
2	1	0	0	1	0	0
3	0	1	1	1	0	0
4	0	0	1	0	0	1

Assume that in all columns p is replaced by proposition 21 and q is replaced by proposition 22. It is impossible that the truth-value of the proposition components differ from one situation to another. The position is like this; even when the same set of propositions with determinate truth-values form the components of various compounds propositions, the truth-value of one compound proposition differs from the truth-value of any other compound propositions. Before we arrive at this conclusion, we must compare the truth-value of component propositions in all possible circumstance.

Even if in one circumstance there is variation in the truth-value, our stand is vindicated. For example, in the table 8, the last two columns possess different truth-values only in the fourth row. Therefore it is clear that in spite of the fact the same set of propositions form components of different compound propositions, the truth-value varies from column to column because besides components, the sentential connective also determines the truth-value of given compound proposition. So the truth-value of a compound proposition is 'uniquely' determined by the truth-value of its components only with respect to that particular compound. However, if we have to explain variation from one column to another, then we also have to consider the role played by sentential connectives. The difference can be aptly summarized in this way; 'vertical variation in truth- value of a compound proposition is a function of the truth-value of components only, whereas horizontal variation is a function of sentential connective' only.

6.5 OTHER FORMS OF COMPOUND PROPOSITION

In the beginning of this unit, it was mentioned that the components of a compound propositions themselves can be compound propositions. We will consider a compound proposition with only three propositions because then we will have eight rows and if there are four propositions we will have sixteen rows. It is because, since any component takes two truth-values (i.e., either true or false), addition of a component would double the number of rows: thus for one component, only two rows as in the case of negation; for two, four rows, as we have seen in other truth table; for three, eight rows; for four, sixteen; for five, thirty two rows, and so on. However, with three simple propositions several compound propositions can be constructed. Therefore it will adequately serve our purpose. The variables and statements are as follows:

25 Alcoholism is a vice . p
26 Courage is a virtue. q
27 Yoga heals diseases. r

Various compound propositions can be constructed out of these propositions. Some of them are considered.

28 (p=>q) Λ (¬q v r)
29 (p =>q) v (p Λ q)
30 (q v r) => p
31 (q => r) v (p Λ r)

It should not be difficult to substitute statements of p, q and r. It is left as an exercise to the students to do the same. There is something more important to clarify. Apart from the fact that the components of propositions 28 to 31 are themselves compound, there are parentheses also. The significance and necessity of parentheses can 13 be easily understood, when compared with simple arithmetic. Compare these two expressions:

i). (5+7)10 = 300

ii). 5+7 x 10 = 75 (1) is false.

It is not even possible to say whether ii) is false or not. Knowing whether a certain expression is true or false is not very significant. But arriving at a determinate expression is significant. This is what exactly parentheses achieve when used appropriately. If they are not used, then it will be a mistake in mathematics, language and logic. Let us consider statement 28 which has four connectives and therefore there are four compound propositions. Though one truth-table is sufficient for our purpose, in order to gain better understanding, we shall split the table:

Table: 9

	p	q	p =>q
1	1	1	1
2	1	0	0
3	0	1	1
4	0	0	1

Table: 10

	r	⌐q	⌐q v r
1	1	0	1
2	0	0	0
3	1	1	1
4	0	1	1

(r v ⌐q is the same as ⌐q v r.) To take next step let us assume that (p => q) is one component and ⌐q v r is another component. Let us transpose columns 3 of Table 9 and Table 10 to Table 11 to compute the result.

Table: 11

p	q	⌐q	r	p => q	⌐q v r	(p => q) Λ (⌐ q v r)

1	1	1	0	1	1	1	1
2	1	1	0	0	1	0	0
3	1	0	1	1	0	1	0
4	1	0	1	0	0	1	0
5	0	1	0	1	1	1	1
6	0	1	0	0	1	0	0
7	0	0	1	1	1	1	1
8	0	0	1	0	1	1	1

Before closing this section one has to learn the method of constructing truth-tables; it is a very interesting part of the study of symbolic logic. Truth-tables are constructed for truth-functions having statement variables that are customarily counted from the middle part of the alphabet like p, q, r, s, … Accordingly, 'Bacon is a writer' is a statement in English; it can be symbolized as 'B'; it can be represented in a variable form as simply 'p'. Before beginning the work of constructing the truth-table we have fix the specific form of the given statement, determine the columns under which the truth-values are to be arranged and limit the number of rows in accordance with number of variables in the specific form of the statement. Let us work with a compound statement: (A=>B) Λ (¬B v C). Its specific form is (p=>q) Λ (¬q v r) Its truth-table is just above (no. 11). [The students are advised to construct truth-tables for the remaining combinations, which are relatively simple. In all cases the number of rows is 8. Since practice makes man perfect, the students are advised to substitute statements for variables in all cases.]

Check Your Progress 2

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1) Define truth-functional logic.

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6.6 LET US SUM UP

Modern logic distinguishes two kinds of statements. All truth-functional propositions are compound. ‘Grammatical’ compound is different from ‘logical’ compound. Truthfunctional compound is a function of sentential connective and truth-values of components. Negation is the simplest (simplest in grammatical sense) form of compound. There are five types of compound propositions, each distinguished by its own set of truthvalues. The truth-values of one compound differ from that of the others at least on one occasion. Difference between implication and bicondition are notable. Components of compound proposition can themselves be compound. To have at least one compound within a compound, we need at least three propositions.

6.7 KEY WORDS

Ambiguity: When a word or a statement carries more than one legitimate meaning it is said to be ambiguous.

Turmoil: Turmoil is a state or condition of extreme confusion, agitation, or commotion.

Main Connective: The connective that determines the basic form of a statement is called main connective. For example, (A=>B) Λ (⌐B v C) is a conjunction whose left hand conjunct is an implication and whose right hand conjunct is a disjunction.

6.8 QUESTIONS FOR REVIEW

1) Distinguish ‘compound’ in grammatical sense from ‘compound’ in logical sense.
2) Bring out the difference and similarity with respect to copula and sentential connective.
3) Define truth-functional logic.

6.9 SUGGESTED READINGS AND REFERENCES

- Basson, A.H. & O'connor, D.J. Introduction to Symbolic Logic. Calcutta: Oxford University Press, 1976.
- Copi, I.M. Symbolic Logic. 4th Ed. New Delhi: Collier Macmillan International, 1973.
- I.M. Copi, Introduction to Logic. 9th Ed. New Delhi: Prentice Hall of India, 1995.
- Kalish, Donold et al. Logic.Techniques of Formal Reasoning. New York: Harcourt Brace
- Jovanovich, 1980.
- Lewis, C.I. & Longford, C.H. Symbolic Logic. New York: Dover Pub. Inc.,1959.
- Suppes, Patricks. Introduction to Logic. New Delhi: Van Nostrand Reinhold, 1957.

6.10 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress 1

1. A compound statement in grammatical sense is independent of its components as far as its truth-value is concerned. However, in logical sense the truth or falsity of compound proposition depends upon the truth or falsity of its components.

2. Both copula and sentential connective perform the function of linking two distinct units; copula links two terms whereas sentential connective links two statements which may be true or false. The number of sentential connectives is always one less than that of statements. The same connective may occur more than once in the given compound proposition. While copula does not determine the truth of combination, the latter determines the same.

Check Your Progress 2

1. Logic which deals with truth-functional compound statements is called truth–functional logic.
2. Implication is false only when the antecedent is true and consequent is false and under all other instances it is true. Bicondition is true only when both the components have the same truth-value, i.e., both components must be true or false together.

UNIT 7: HISTORY AND UTILITY OF SYMBOLIC LOGIC

STRUCTURE

7.0 OBJECTIVES

In this unit, an attempt is made to present a history of symbolic logic. You will be quick enough: to notice that the moment you enter symbolic logic, you are confronted with mathematics as well.

- To learn that development of logic and mathematics are inseparably related.
- To know that logic and mathematics are two components of one enterprise.
- To be familiar with conceptual developments with a brief description of what they are.
- To set your priorities right, to identify the elements of logic in mathematical discussions.

7.1 INTRODUCTION

History of logic can safely be divided into three phases; ancient logic, medieval logic and modern logic. It is necessary to bear in mind that one is not just replacement for the other and that elements of later phase can be discerned in the earlier phase. Therefore development is significantly in terms of correction and improvements, but not total rejection. Therefore it is absolutely necessary to admit that the limitations of ancient and medieval systems of logic paved way for the rise of symbolic logic and its value in addition to pioneering work by some mathematicians.

The history of logic deals with the study of the development of the science of valid inference (logic). Formal logics developed in ancient times in India, China, and Greece. Greek methods, particularly Aristotelian logic (or term logic) as found in the Organon, found wide application and acceptance in Western science and mathematics for millennia. The Stoics, especially Chrysippus, began the development of predicate logic.

Christian and Islamic philosophers such as Boethius (died 524), Ibn Sina (Avicenna, died 1037) and William of Ockham (died 1347) further developed Aristotle's logic in the Middle Ages, reaching a high point in the mid-fourteenth century, with Jean Buridan. The period between the fourteenth century and the beginning of the nineteenth century saw largely decline and neglect, and at least one historian of logic regards this time as barren. Empirical methods ruled the day, as evidenced by Sir Francis Bacon's Novum Organon of 1620.

Logic revived in the mid-nineteenth century, at the beginning of a revolutionary period when the subject developed into a rigorous and formal discipline which took as its exemplar the exact method of proof used in mathematics, a hearkening back to the Greek tradition. The development of the modern "symbolic" or "mathematical" logic during this period by the likes of Boole, Frege, Russell, and Peano is the most significant in the two-thousand-year history of logic, and is arguably one of the most important and remarkable events in human intellectual history.

Progress in mathematical logic in the first few decades of the twentieth century, particularly arising from the work of Gödel and Tarski, had a significant impact on analytic philosophy and philosophical logic, particularly from the 1950s onwards, in subjects such as modal logic, temporal logic, deontic logic, and relevance logic.

7.2 EARLIEST CONTRIBUTIONS TO LOGIC

The greatest contribution of Aristotle to logic, undoubtedly, is his theory of syllogism in which the theory of classes and class relation is implicit. Another significant contribution of Aristotle is his notion of variables. Classes themselves are variables in the sense that in any proposition subject and predicate terms are not only variables but also they are the symbols of classes.

Finally, the class relation, which is explicit in his four-fold analysis of categorical proposition, is understood as inclusion or exclusion - total or partial. Theophrastus, a student of Aristotle, developed a theory of pure hypothetical syllogism. A hypothetical syllogism is said to be pure if all the three propositions are hypothetical propositions. Theophrastus showed that pure hypothetical inference (an inference which consists of only hypothetical propositions) could be constructed which corresponds to inference consisting of only categorical propositions (which Aristotle called syllogism). A school of thought flourished during Socrates' period known as Megarians. The first generation of Megarians flourished in the 5th century B.C. onwards. In the 4th century B.C. one Megarian by name Eubulides of Miletus introduced now famous paradox – the paradox of liar. The last Greek logician, (who is also 'lost' because none of his writings is extant), who is worthy of consideration is Chrysippus of whom it is said that even gods would have used the logic of Chrysippus if they had to use logic. Peter Abelard, who lived in the 11th Century, is generally regarded as the first important logician of medieval age followed by William of Sherwood and Peter of Spain in the 13th Century. They continued the work of Aristotle on categorical proposition and syllogism and other related topics. In reality, no vacuum was created in medieval age and hence there was continuity from Aristotelian logic to modern logic though no original contribution came from any logician. The most notable contribution to logic in this period consists in the developments, which took place in several important fields like analysis of syntax and semantics of natural language, theories of reference and application, philosophy of language, etc., the relevance of which was, perhaps realized only very recently. These are precisely some of the topics of modern logic. William of Sherwood and Peter of Spain were the first to make the distinction between descriptive and nondescriptive functions of language. They reserved the word 'term' only for descriptive function. Accordingly, only subject and predicate qualify for descriptive function and hence in categorical proposition we can find only two terms. These were called categorematic whereas other components of a sentence like 'all, some, and no', etc. were called syncategorematic. The former are terms whereas the latter are only words. Hence, terms were regarded as special words. It is in this context that the medieval logicians made semantic distinction of language levels. Categorematic term was divided into two classes, terms of first intension and terms of second intension. First class stands for things whereas the second stands not for a thing but for a language sign. In a limited sense, and at elementary level, it can be said that subject represents first class and predicate represents second class. Another field covered by medieval logicians was that of quantification which is of great importance in modern logic. again, this is another important topic of modern logic.

7.3 LIMITATIONS OF ARISTOTELIAN LOGIC

The very fact that Aristotle constructed an extraordinarily sound system of logic became its nemesis. Just as Newtonian Physics was held as infallible for a little more than two hundred years, Aristotle was held on similar lines for nearly two thousand years. However, neither of them anticipated this treatment to their systems. While this is one reason for the delayed beginning of modern logic, second and the most important reason is that mathematics also had not yet been developed. The emphasis is not upon the defects of the system, but on the limitations because, ironically, the defects did not hinder the growth of logic. It may also be true that had the defects been detected very early, situation would not have been much different because time was not ripe for take-off of symbolic logic. One serious limitation of Aristotelian system is its narrow conception of proposition. He restricted it to subject-predicate form. Though class-relation is implicit in this theory of syllogism, Aristotle ignored it. There is little wonder that Aristotle did not think of any other relations. Consider these two examples:

All men are mortal.
All mortal beings are imperfect
All men are imperfect.
Bangalore is to the east of Mangalore.
Madras is to the east of Bangalore.
Madras is to the east of Mangalore.

Both these arguments are valid in virtue of transitive relation. Aristotle recognized only the first example as valid and what is surprising is that he considered only the first type as an argument. The result is that most of the mathematical statements ceased to be propositions in his analysis. His narrow outlook eliminated any possibility of logic and mathematics interacting. Consequently, considerable types of arguments with much complicated structure fall outside the limits of Aristotelian logic and hence remain unexamined. Medieval logic, in spite of remarkable contributions to logic, did not take logic a step ahead because whatever research was done was only an in-house work, i.e., work within the system. What was required was transition from one system to another. In what sense modern logic makes progress over Aristotelian logic? It is very important to answer this question. Modern logic did not supersede Aristotelian logic in the sense in which an amendment to constitution results in one act replacing another. Modern logic neither superseded nor succeeded Aristotelian logic. It only extended the boundaries of the system. Existing rules remained not only acceptable but also were augmented by new set of rules. Later we will learn that among nine rules of inference, six are from Aristotelian logic. And simple conversion and observation were retained but given ‘extended meaning’ in terms of the rules of commutation and double negation respectively. Meaning was extended because logic and mathematics mutually made inroads into one another’s territory. In a similar fashion, the use of variables also underwent a change. While Aristotle used variables only to represent terms, modern logic extended the use to propositions as well. This inclusion had far reaching consequences. Lastly, quantification, which was introduced during medieval age, was further improvised. The foregoing discussion should make one point clear. The tools used to test arguments or to construct arguments by Aristotelian system are insufficient. Modern logic further augmented the 4 tools not only in number but also in variety. It should be remembered that the sky is the limit to improve and add. Before we enter the modern era, one interesting question must be considered. How should we explain the relation between logic and mathematics? Two philosophers have differently described this relation. Raymond Wilder says that for Peano and his followers ‘logic was the servant of mathematics’. Wilder put it in a more respectable and acceptable form, in connection with Frege's philosophy of mathematics, ‘dependence (of mathematics) on logic… was more like that of child to parent than servant to master. Basson and O’connor have echoed more or less similar views while relating classical logic to modern logic. It is like embryo related to adult.

7.4 HISTORY AND UTILITY OF SYMBOLIC LOGIC

At this stage, two aspects must be made clear. Modern logic is also called symbolic logic because symbols replaced words to a great extent. Second, symbolic logic and mathematics do not stand sundered; so much so, modern logic is also called mathematical logic, which A.N. Prior terms ‘loosely called.’ However, Prior’s remark has to be taken with a pinch of salt. Very soon, we realize that almost all people, whose names are associated with symbolic logic, are basically mathematicians. And at some stage it becomes extremely difficult to separate logic from mathematics and, if attempted, it will be an exercise in futility. However, a definite limitation must be considered. When we talk of mathematics we talk of pure mathematics only. So when we deal with history of a symbolic logic we deal with the history of pure mathematics. Where exactly does symbolic logic score over classical logic? Language is, generally, ambiguous. It is so for two reasons. In the first place, a significant number of words are equivocal and secondly, many times the construction of sentences and their juxtaposition are misleading so much so they convey meaning very different from what the speaker or author intends. Replacement of words by symbols and application of logical syntax different from grammatical syntax completely eliminates ambiguity. The meaning of logical syntax becomes clear in due course when sentences are represented by symbols. It is possible to test the validity of arguments only when the statements are unambiguous. Further, use of symbols saves time and effort required to test the validity of arguments.

7.5 THE RISE OF SYMBOLIC LOGIC

Generally, bibliography of symbolic logic compiled by Alonzo Church is reckoned as authentic to determine the beginning of symbolic logic. In the year 1666, Leibniz published (or wrote) a thesis on a 'Theory of Combinations titled 'Dissertatio de Arte Combinatoria.' It is said that the beginning of symbolic logic coincides with this work. If so, Chrysippus has to be heralded as the forerunner of symbolic logic because according to records long before Leibniz he showed some interest in Combinations. So he must have done some work on Combinations, which was, further, followed up by some logicians in the thirteenth century. In brief, let us describe the subject-matter of Combinations. Leibniz was more concerned with such issues as semantic interpretations of logical formulas. One example may clarify semantic consideration or considerations which engaged Leibniz. What does the statement 'All men are mortal' mean? Does it mean that every member of the class of men is also a member of the class of mortal beings? Or does it mean that every man possesses the attribute of being a mortal? Or does it mean that the attribute of 'being man' includes the 'attribute of being mortal'. In other words, the focus of this consideration is on the choice between extensional approach and intentional approach. Class-membership issue is extensional whereas attribute-inclusion or attribute – exclusion is intentional. Another notable contribution of Leibniz was his work on logical algebra or logical calculus, which consists of several experimental sorts of studies. Some laws, which are features of his study, are laws of identity and explicit statement of transitive relation, which made Aristotelian syllogism significant. Consider these two rules:

a b is a

ab is b

These rules become intelligible when we substitute terms for a & b. suppose that a = intelligent;

b = man

1) Intelligent man is a man

2) Intelligent man is intelligent

Likewise consider another rule:

if a is b and a is c then a is bc.

Again substitute of, b and c, a = Indian, b = Asian, c = Hindu. Then 3 becomes

If Indian is an Asian and Indian is a Hindu, then Indian is an Asian Hindu.

An important requirement of logical algebra is that substitution must be possible; this particular relation was explicitly recognized by Leibniz. In the 18th century two mathematician, Euler and Lambert contributed to the development of logic. While Euler is known for geometrical representation of propositions through his circles, Lambert developed logical calculus on intensional lines. For example, if a and b are two concepts, then a + b becomes a complex concept and ab stands for conceptual element common to a and b. What applies to class membership applies also to attributes. Bolzano is another logician who contributed to logic in the 19th century. He regarded terms and propositions as fundamental constituents of logic. He is known for an extraordinary approach to the logical semantics of language. In this context, he regarded propositions as having universal application when certain conditions are satisfied and as universally inapplicable under certain other conditions and as consistent under certain other conditions. Bolzano in fact, modified Kant's definition of 'analytic judgment' using this particular criterion. Another important contribution of Bolzano was his conception of probability. He introduced some modifications into Laplace's conception of probability, which was widely held during his time. Laplace defined probability as equipossible while determining the probability value when only two possibilities are available as in the case of tossing of the coin. In fact, Bolzano's modification avoids this particular element. This is crucial because 'equipossible' involves circularity. By avoiding this term, Bolzano could avoid circularity, which was inherent in Laplace's theory. In 1847 two mathematicians, de Morgan and George Boole published 'Formal Logic' and 'The Mathematical Analysis of Logic respectively. Symbolic logic actually took off from this point of time. De Morgan gave to the world of logic now famous notion of complement which was later 6 exploited by John Venn to geometrically represent distribution of terms and test syllogistic arguments. De Morgan showed that if there are two classes, then there are four product classes and Jevons showed that if there are three classes, then there are eight product classes. So generalizing this relation, we can say that the relation between the number of classes and the number of product classes is given by the formula, n = 2x . Where, 'n' stands for the number of product-classes and x stands for the number of terms. This formula is only indicative of the type of relation, which holds good between classes (or sets) and product classes because there is no syllogism with more than three terms and no proposition (in traditional sense) has more than two terms. He also gave a formula known as de Morgan's law to write the contradiction for disjunctive and conjunctive propositions. Boole's contribution to the rise of symbolic logic far exceeded that of any other logicians considered so far. He conceived the idea that the laws of algebra do not stand in need of any interpretation. This idea led Boole to describe these laws as calculus of classes in extension. In 1854 he published another work 'An Investigation of the

Laws of Thought!' It is in this work that the germs of the 20th century symbolic logic can be traced. While Lambert invented union of concepts on intensional analysis. Boole invented union of sets on extensional basis. He used '1' to designate the universe. Following de Morgan, Boole called it the universe of discourse. He introduced the following laws, which play crucial role in mathematical logic.

1 Union of any set and universal set is a universal set. Let X be a set. Then 1+X =1

2 Product of a universal set and any non-null set X is X itself.

3 Product of null-set and any non-null set (universal set included) is a null-set itself.

If X is a non-null set, the 1 – X is its complementary.

5 It is self-evident that product of any non-null set and its complementary is a null-set.

5 Stands for Boole's definition of contradiction. He also showed that if X, Y, Z,…etc. stand for non- null sets, then all laws of algebra hold good. Most important among them are what are known as distributive and commutative laws. For the sake of brevity, these laws are stated as follows:

1 Distributive Law: a(b+c) = ab + ac

2 Commutative Law: ab= ba

or a+b=b+a

Using the concept of complementary class, Boole also showed that 'A, E, I and O' of traditional logic can be reinterpreted. His suggestion was geometrically represented by Venn. In this interpretation, Boole actually considered what is called class logic, which later became the cornerstone of set theory. In logic, there is another topic called calculus of propositions. Boole integrated these two and defined the truth-value of what are called compound propositions which also consist of variables. While in the first interpretation the variables represent the sets or terms, in the second interpretation they represent the propositions.

Consequently, products of classes, here, become conjunction and union or addition of classes becomes disjunction. Complement of a set becomes negation of a proposition. 7 Boolean analysis of logic is also called Boolean algebra for two reasons. In the first place, he freely used variables to explain various aspects of logic. Extensive use of variables characterizes algebra. Secondly, he defined all four operations of algebra; addition, multiplication, subtraction and division and extended the same to logic. Venn's contribution to logic was partially mentioned earlier. Therefore the remaining part requires to be mentioned. Venn is well-known for making qualitative distinction, in addition to traditionally held quantitative distinction between universal and existential (particular) which has far reaching consequences. The distinction is that while universal proposition (in modern logic universal quantifier) denies the existence of membership in a class, existential quantifier affirms the same. Secondly, a large number of deductive inferences became invalid as a result of this description. The irony is that in this situation, progress is marked not by augmentation but by depletion in the number of inferences. There were certain anomalies in Boolean system. Consider two identical sets, say X and Y where every member of X is a member of Y and every member of Y is a member of X; for example, the class of bachelors and the class of unmarried men. The product class should yield X Y. Since Y = X, XY=X2 or Y2 . In algebra it makes sense, but surely not in logic. Similarly X+Y, the union of two sets ought to become 2X. Again, it holds good in algebra but not in logic. Jevons, a student of de Morgan, succeeded in eliminating these anomalies; according to his interpretation, the union of two identical sets does not double the strength, say from n to 2n. The reason is simple; every member is present in both the sets. We cannot count one individual as two just because he or it is present in two sets simultaneously. The same reasoning applies to product of identical sets. If there are 100 bachelors and 100 unmarried men then the product of these two sets does not produce 1002 = 10,000 bachelors who are also unmarried men, but 100 only. C.S. Peirce resolved this anomaly in a different way. He identified logical addition with inclusive or instead of exclusive or (either p or q but not both is an example for exclusive or and either p or q or both is an example for inclusive or). Peirce introduced a symbol ⊃ for class inclusion. He strangely argued that there is no difference between a proposition and inference or implication. In the ultimate analysis only implication survives. Secondly, all implications have quantifiers, which may be explicit or implicit. While Peirce thought that implication is the primary constituent of logic, at a later stage, there were attempts to eliminate implication and retain only negation and conjunction. While introducing symbols in a set of formulas Peirce was driven by a definite motive. He believed that symbols should resemble what they represent say thoughts. To achieve his aim, Peirce used, what he called, ‘existential graphs’. They were not graphs in geometrical sense. He regarded parentheses themselves as graphs. For example, 'if p, then q'was represented graphically, by Peirce by using parentheses. He inserted p and q within parentheses and represented as (p (q)). Christine Ladd Franklin invented a new technique of testing syllogism called antilogism or inconsistent triad. In addition to, Venn’s diagram, antilogism also eliminated weakened and strengthened moods on the ground that particular propositions cannot be deduced from universal propositions only.

Gottlob Frege is one of the pioneers, who gave a new dimension to mathematical logic. In 1879 ‘Begriffsschrift’ the first of his most important works was published followed by Die Grudlagan der Arithamatik in 1884. His first work dealt with proper symbolization with the help of rules of quantification. His intention was to codify logical principles used in mathematical reasoning like substitution, modus ponens, etc. In this work he introduced the notion of function, which was later renamed as propositional function. He also introduced a system of basic formulas for propositions in terms of implication and negation. In his second work, Frege made the most crucial attempt to trace the roots of mathematics to logic. He himself regarded arithmetic as simply a development of logic. Consequently, every proposition of arithmetic became merely a law of logic. History has recorded that Frege’s thesis would not have got what it deserved but for Russell’s discovery of Frege. Hence the relation between arithmetic and logic is known as Frege-Russell thesis. It is said that modern logic began with Frege. It means that in one sense the history of symbolic logic stops before Frege. Whatever development that took place after Frege's period characterize contemporary logic. Even in this period, there were remarkable changes with new theses being presented regularly. Giuseppe Peano tried to establish the relation between logic and mathematics in a slightly different manner. Instead of tracing the roots of mathematics to logic, Peano tried to express mathematical methods in a different form similar to that of logical calculus. For example, the successor of 'a' was designated by the symbol 'a+'; also in addition to the symbol ⊃ he introduced another symbol ∈. This shows that implication or class inclusion (⊃) is distinct from 'element of' or 'belongs to'. In Peano's system there is no interpretation of any symbol and hence mathematics becomes a formal system. In the beginning of the 20th century Zermelo proposed his theory of sets known as Axiomatic Set theory. He intended his theory to be free from contradictions. He regarded it as well ordered because it was axiomatized. His claim was totally rejected by Poincare. Perhaps only two mathematicians disputed the theory that mathematics has its foundations in logic. Opposition to this approach developed first in the 19th century. Kronecker, a professor of mathematics at the University of Berlin in 1850s, was the first mathematician to oppose this dominant trend. He disagreed with Cantor's theory of sets which included the concept of infinity. Kronecker went to the extent of arguing that integers are made by God, but everything else is the work of man. After Kronecker, it was Poincare who believed that mathematics does not have its base in logic. His main thesis is that in the first place, mathematical induction cannot be reduced to logic; secondly, according to him, even mathematics proceeds from particular to universal only; a clear opposition to deductive logic.

7.6 THE AGE OF PRINCIPIA MATHEMATICA (PM)

In 1910 Bertrand Russell in association with A.N. Whitehead published Principia Mathematica. What was referred to as the Frege-Russel thesis in the previous section found exposition in this work. Only a few aspects of this great work can be dealt here. The principal thesis remains the same, that mathematics is an extension of logic. Jevons, earlier, remarked that 'algebra' is nothing but highly developed logic' to which Frege added: 'inferences….. are based on general laws of logic.' Frege was actually referring to mathematical induction. In the preface itself the 9 authors admitted that 'thanks to Peano and his followers symbolic logic… acquired the technical and the logical comprehensiveness that are essential to a mathematical instrument'. Clearly, the new age mathematicians bypass Poincare and Kronecker in this regard. PM makes a clear distinction between proposition and propositional function. While variables constitute propositional function, substitutions to variables constitute propositions. The former is neither true nor false. But the latter is either true or false. For example, X is the husband of Y is neither true nor false. But Rama (X) is the husband of Sita (Y) is true. A key logical term, which finds place in PM is material implication. Russell and Whitehead used '⊃' to designate implication. Material implication is defined as follows: $p \supset q \equiv \sim p \; q \vee$ Truth-values were assigned by PM as follows. Both p and q can be true together, or when p is false, q may be false or true. But when p is true q cannot be false. Implication, therefore, does not imply necessary connection. To distinguish implication from prohibited possibility Russell and Whitehead used material implication instead of mere 'implication'. This particular definition of material implication has a very important consequence. 'Necessary relation' was an unwanted metaphysical baggage, which was overthrown by Hume. But there was no way of interpreting implication in the absence of necessary relation. Fixation of truthvalue by PM made a distinct advance in this case. And it is precisely this type of implication that is used in mathematics. Consider a very familiar example, 'If ABC is a plane triangle, then the sum of the three angles equals two right angles'. That there is no plane triangle at all does not affect the relation because even when the antecedent is false the consequent can continue to be true. Hence it comes to mean that a true premise can imply only true conclusion whereas a false premise can imply either true or false conclusion. PM includes five axioms (Russell and Whitehead use the word 'principle'), which can be regarded as primitive logical truths. They are follows:

1 Tautology (Taut)

2 Addition (Add)

3 Permutation (Perm)

4 Association (Assoc)

5 Summation (Sum)

Example provided here is taken from the text itself. The authors in all these cases use the symbol I- which is read 'it is asserted that' or it is true that' and the dots after assertion I– sign indicate range. 'v' is read 'or' and '⊃ is read 'if...then'.

Taut: I-: p p. ˅ ⊃ .P It is true that p or p implies p.

Add: I-: q. ⊃ . p q It is true that if q, then p or q. ˅

Perm: I-: p q. ˅ ⊃ .q p If p or q, then q or p. ˅

Assoc: I-: p ˅ (q ˅ r). ⊃. q ˅ (p˅ r) If p or q or r, then q or p or r.

Sum: I- :q ⊃ r. ⊃: p q. ˅ ⊃ .p r If q implies r, then p or q implies p or r.

For 'Add' the example is 'if today is Wednesday (q), then today is either Tuesday or Wednesday. The examples can be constructed on similar lines for other axioms. For perm, the example read as follows; if today is Wednesday or Tuesday, then today is Tuesday or Wednesday. In all cases, the sentences are preceded by 'it is true that'. The colleen immediately after the assertion sign indicates range, but the dots which follow or precede variables are only customary. PM also includes equivalence relation, which explains the equivalence of the law of the Excluded Middle and the Law of contradiction. In the beginning of the summary of *3 the authors say that 'it is false that either p is false or q is false, which is obviously true when and only when p and q are both true. Symbolically, p.q = . ~ (~ p˅ ~ q) Reductio ad absurdum is one method accepted by mathematics. It means that the contradiction of what has to be proved is assumed to be true and then the conclusion contradicting the assumption is deduced. This contradiction shows that the assumption is false in which case its contradiction must be true. This is again a primitive logical truth. The principle of double negative is another, which can be easily derived from the law of the Excluded Middle. David Hilbert contributed to the development of logic which led to the birth of what is known as metamathematics. His theory of mathematics is known as formalist theory of mathematics. This theory of mathematics makes a distinction between sequence and statement. It asserts that a sequence is neither true nor false. This distinction corresponds to the one made in classical logic between a sentence and a proposition. An important aspect of metamathematics is its axiomatic approach. A system, be it mathematics or anything else, can be formalized only when axiomatic method is followed. A system is said to be formalized or axiomatized only when all propositions in the system stand in a definite logical relation. Consistency is one such relation. Therefore, a consistent system, in Hilbert's analysis is an axiomatized system. A distinguishing mark of Hilbert's analysis is his 'discovery' of 'ideal limit'. From the days of Cantor and Weirstrass who introduced the concept of 'infinity' or 'transfinite' the concept of ideal limit engaged the attention of mathematicians. While elementary number theory could be empirically interpreted, infinity could not be interpreted in that manner. So Hilbert chose to regard transfinite as limit. There should not be break in history – circuit. Therefore another contribution of Hilbert secures a place in our discussion. Hilbert embarked upon his project to defend classical mathematics from one theory of mathematics known as intuitionism spearheaded by the Dutch mathematician Jan Brouwer, according to whom mathematics is not a system of formulas but is a sort of abstract activity, which abstracts the concept of 'numberness.' By any standard, 'intuitionist mathematics ceases to be a logical enterprise, but confines itself to the narrow domains of psychological activity at best and some sort of esoteric activity at worst. Following the tradition of PM, Emil Post presented the method of truth-tables published as 'Introduction to a General Theory of Propositions' in the American Journal of Mathematics in 1921. In this paper, Post included not only classical logic, which allowed only two values but a system allowing many values. In the same year Wittgenstein's Tractus logico-Philosophicus was published, which also included this technique. Wittgenstein held the view that mathematics is nothing but a bundle of tautologies. While this is the view of earlier Wittgenstein, in later Wittgenstein the conception of mathematics underwent dramatic change. In 'Remarks on the Foundations of Mathematics' Wittgenstein argues that both logic and mathematics form parts of language games. At this point of time he became a conventionalist and argued that mathematical propositions are immune to falsification. This position of Wittgenstein is much closer to intuitionism than to anything else. Rudolph Carnap's contribution to symbolic logic consists in the extension of the same to epistemology and philosophy of science. He argued that all meaningful sentences belong to the language of science. He followed what is called the 'principle of tolerance' with which any form of expression could be defended if sufficient logical rules are there to determine the use of such expression. Under the influence of Alfred Tarski, he included such notions as truth and meaning in his analysis. Kurt Goedel is another important philosopher of mathematics. He was concerned with intuitionistic and classical mathematics equally. He is widely known for his famous 'Incompleteness Theorem'. He showed that it is impossible to prove consistency of certain formulations of arithmetic by methods which are internal to the system. He showed that what is provable in classical mathematics is also provable in intuitionist mathematics. The only requirement is that what has to be proved must be properly interpreted. Alonso Church is a noted historian of symbolic logic. Logicians and mathematicians alike are interested in questions related to the decidability of logical and mathematical theories. His main thesis is that there is no general technique to determine or discover the truth or proof of any proposition in arithmetic. In this respect, Church stands opposed to Hilbert who argued that classical mathematics is a consistent system. W.V.O Quine and Curry are two other prominent personalities. While Quine is known for his contribution to the development of set theory, Harkell B. Curry's name is associated with a new branch of logic called 'Combinatory Logic'. It had its birth in H.M.Shaffer's discovery of 'stroke' symbol (I) with which all sentential connectivity could be interpreted. This was extended by Moses Schonfinkel to quantifiers also. Stroke symbol was introduced to simplify the use of symbols and subsequently Schonfinkel extended it to eliminate variables. Curry proceeded further with Schonfinkel's works with set of operations different from stroke symbol. He introduced what is called the theory of λ – conversion (λ is read 'lamda'), where λ is known as binary operation. Church used this operation to analyze formal systems to which variables belong and to which arbitrary objects can be substituted. Here objects mean the functions in which they stand for arguments. It means that a variable in a system is substituted by an argument. λ – conversion is a theory proposed by Church in connection with such substitutions. In short, symbolic logic is a system of algebraic combination and mechanical substitution of symbols for the purpose of inference. It is the study of symbolic abstractions that captures the formal features of logical inference. C.I. Lewis observes the following characteristics for symbolic logic: the use of ideograms (i.e., signs that stand directly for concepts) instead of phonograms (signs that depict sounds first and indirectly concepts); deductive method and use of variable having definite range of significance. It has mainly two parts: truth-functional or propositional or sentential logic and predicate logic. The former is a formal system in which 12 propositions can be formed by combining simple propositions using sentential connectives, and a system of formal proof in determining the validity of arguments. Predicate logic provides an account of quantifiers in the symbolization of arguments and laws for the determination of their validity.

Check Your Progress 1

Note: a) Use the space provided for your answer.

b) Check your answers with those provided at the end of the unit.

1. Examine Boole's contribution to modern logic.

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2. Examine the role played by PM in the 20th century logic.

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3. Contrast Hilbert's and Goedel's views on proofs in mathematics.

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4. What is the significance of Shaffer's and Schonfinkel's studies? Explain.

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7.7 LET US SUM UP

Logic has its roots in Greek civilization. Aristotle systematized the technique of thinking. During medieval ages, lot of research work was undertaken within the limits of Aristotelian system. Modern logic took its birth with Leibniz' work 'Dissertatio de Arte Combinatoria'. Boole's works provided impetus to the growth of symbolic logic. Contemporary symbolic logic begins with de Morgan. Initially, Frege and Russell and later, Russell and Whitehead heralded a new era in symbolic logic. Combinatory logic has its beginning in H.M. Shaffer's work which was later developed by Haskell B. Curry. Today logic and mathematics have become two faces of the same coin.

7.8 KEY WORDS

Theorem: In mathematics, a theorem is a statement proved on the basis of previously accepted or established statements such as axioms.

7.9 QUESTIONS FOR REVIEW

1. Examine Boole's contribution to modern logic.
2. Examine the role played by PM in the 20th century logic.
3. Contrast Hilbert's and Goedel's views on proofs in mathematics.
4. What is the significance of Shaffer's and Schonfinkel's studies? Explain.

7.10 SUGGESTED READINGS AND REFERENCES

- Basson, A.H. & Connor, D.J.O. Introduction to Symbolic Logic. Calcutta: Oxford University Press, 1976.
- Edwards, Paul, ed. Encyclopedia of Philosophy. Vol 4. Macmillan and free Press, 1972.
- Lewis, C. I. A Survey of Symbolic Logic. New York: Dover Publication, 1960
- Wilder L., Raymond and Wiley. Introduction to the Foundations of Mathematics.
- New York: John and Sons Inc, 1952.
- Alexander of Aphrodisias, In Aristotelis An. Pr. Lib. I Commentarium, ed. Wallies, Berlin, C.I.A.G. vol. II/1, 1882.
- Avicenna, Avicennae Opera Venice 1508.
- Boethius Commentary on the Perihermenias, Secunda Editio, ed. Meiser, Leipzig, Teubner, 1880.
- Bolzano, Bernard Wissenschaftslehre, (1837) 4 Bde, Neudr., hrsg. W. Schultz, Leipzig I-II 1929, III 1930, IV 1931 (Theory of Science, four volumes, translated by Rolf

George and Paul Rusnock, New York: Oxford University Press, 2014).

- Bolzano, Bernard Theory of Science (Edited, with an introduction, by Jan Berg. Translated from the German by Burnham Terrell – D. Reidel Publishing Company, Dordrecht and Boston 1973).
- Boole, George (1847) The Mathematical Analysis of Logic (Cambridge and London); repr. in Studies in Logic and Probability, ed. R. Rhees (London 1952).
- Boole, George (1854) The Laws of Thought (London and Cambridge); repr. as Collected Logical Works. Vol. 2, (Chicago and London: Open Court, 1940).
- Epictetus, Epicteti Dissertationes ab Arriano digestae, edited by Heinrich Schenkl, Leipzig, Teubner. 1894.
- Frege, G., Boole's Logical Calculus and the Concept Script, 1882, in Posthumous Writings transl. P. Long and R. White 1969, pp. 9–46.
- Gergonne, Joseph Diaz, (1816) Essai de dialectique rationelle, in Annales de mathématiques pures et appliquées 7, 1816/7, 189–228.
- Jevons, W.S. The Principles of Science, London 1879.

7.11 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress 1:

1. Boole’s contribution to the rise of symbolic logic far exceeded that of any other logicians considered so far. He conceived the idea that the laws of algebra do not stand in need of any interpretation. This idea led Boole to describe these laws as calculus of classes in extension. In 1854 he published another work ‘An Investigation of the Laws of Thought!’ It is in this work that the germs of the 20th century symbolic logic can be traced.

2. The publication of Principia Mathematica by Russell and Whitehead heralded a new era in the history of mathematics and logic. In this work they established that logic is the foundation of mathematics. The term implication acquired a new meaning when new rules of inference were evolved. These rules of inference forced logicians to distinguish implication from entailment. Also this work influenced Emil Post to present the methods of truth-table which is the backbone of mathematical logic. The earlier Wittgenstein was also partly influenced by this work.

3. David Hilbert contributed to the development of logic which led to the birth of what is known as metamathematics. His theory of mathematics is known as formalist theory of mathematics. This theory of mathematics makes a distinction between sequence and statement. It asserts that a sequence is neither true nor false. This distinction corresponds to the one made in classical logic between a sentence and a proposition. An important aspect of metamathematics is its axiomatic approach. A system, be it mathematics or anything else, can be formalized only when axiomatic method is followed. A system is said to be formalized or axiomatized only when all propositions in the system stand in a definite logical relation. Consistency is one such relation. Therefore a consistent system, in Hilbert's analysis is an axiomatized system. Kurt Godel is another important philosopher of mathematics. He was concerned with intuitionistic and classical mathematics equally. He is widely known for his famous 'Incompleteness Theorem'. He showed that it is impossible to prove consistency of certain formulations of arithmetic by methods which are internal to the system. He showed that what is provable in classical mathematics is also provable in intuitionist mathematics. The only requirement is that what has to be proved must be properly interpreted.

4. Combinatory logic had its birth in H.M.Shaffer's discovery of 'stroke' symbol (I) with which all sentential connectivity could be interpreted. This was extended by Moses Schonfinkel to quantifiers also. Stroke symbol was introduced to simplify the use of symbols and subsequently Schonfinkel extended it to eliminate variables.